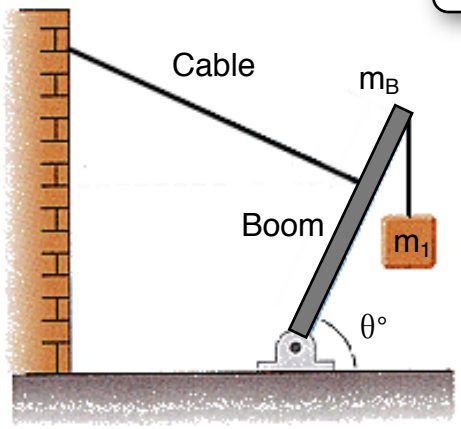
Physics 195 Chapter 12 Problem 40

A 1350 kg uniform boom is supported by a cable. The length of the boom is *l*. The cable is connected 1/4 the way from the top of the boom. The boom is pivoted at the bottom, and a 2250 kg mass hangs from its top. The angle between the ground and the boom is 55°. Find the **tension** in the cable and the **components of the Normal force** on the boom by the floor.

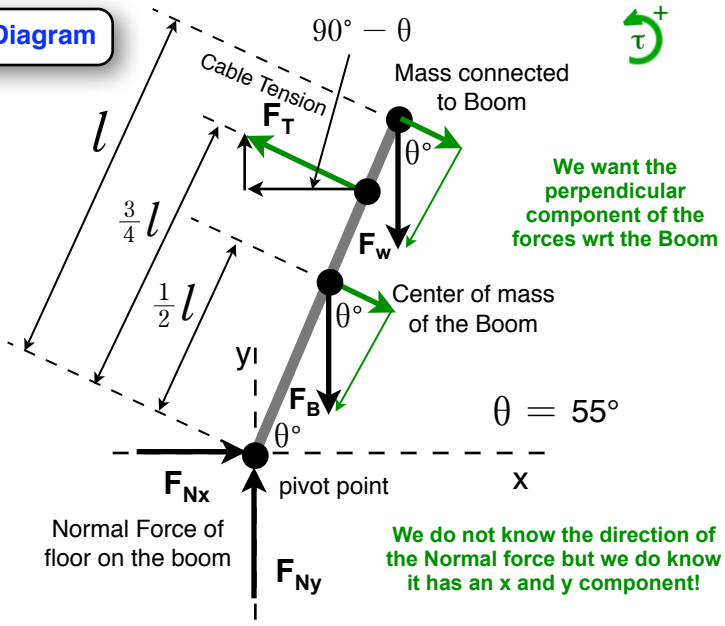


Static Equilibrium

$$\tau = F_\perp d = I\alpha = 0$$

FreeBody Diagram

A)



We want the perpendicular component of the forces wrt the Boom

We do not know the direction of the Normal force but we do know it has an x and y component!

B) Find the tension in the cable.

We want the perpendicular component of the forces wrt the Boom

The Normal Force acts on the pivot so the torque is zero.

Sum of Torques

$$\Sigma\tau = -F_B\cos\theta\left(\tfrac{1}{2}l\right) + F_T\left(\tfrac{3}{4}l\right) - F_W\cos\theta\,(l) = I\alpha = 0$$ *Static Equilibrium*

The force due to the center mass and the weight are causing a negative torque.

$$F_T\left(\tfrac{3}{4}l\right) = F_B\cos\theta\left(\tfrac{1}{2}l\right) + F_W\cos\theta\,(l)$$

$$F_T = \frac{F_B\cos\theta\left(\tfrac{1}{2}l\right) + F_W\cos\theta\,(l)}{\left(\tfrac{3}{4}l\right)} = \left(\tfrac{2}{3}\right)F_B\cos\theta + \left(\tfrac{4}{3}\right)F_W\cos\theta$$

$$F_T = \left(\tfrac{2}{3}\right)1350kg\,(9.8m/s^2)\cos 55° + \left(\tfrac{4}{3}\right)2250kg\,(9.8m/s^2)\cos 55° = \underline{21,900N}$$

C) Find the components of the Normal Force.

Sum of Forces Look for ALL the Forces pointing in the x or y direction on your diagram

$$\Sigma F_x = F_{Nx} - F_T\cos(90° - \theta) = 0$$ *Static Equilibrium*

$$F_{Nx} = F_T\cos(90° - \theta) = 21,900N\cos(90° - 55°) = \underline{17,900N}$$

$$\Sigma F_y = F_{Ny} + F_T\sin(90° - \theta) - F_B - F_W = 0$$ *Static Equilibrium*

$$F_{Ny} = -F_T\sin(90° - \theta) + F_B + F_W$$

$$F_{Ny} = -21,900N\sin(90° - 55°) + (1350kg + 2250kg)\,9.8m/s^2 = \underline{22,700N}$$

The Normal Force

$$F_N = \sqrt{(17,900N)^2 + (22,700N)^2} = \underline{28,900N} \quad \theta = \tan^{-1}\frac{22,700N}{17,900N} = \underline{51.7°}$$