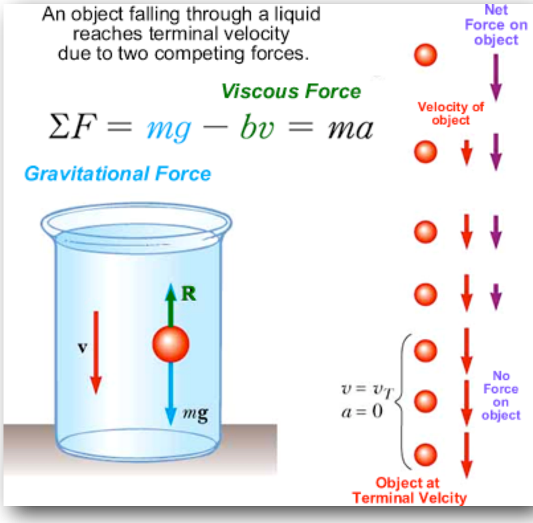
A Mechanical Analog



Sum of the Forces

$$\Sigma F = mg - bv = ma$$

Solve for the acceleration

$$g - \frac{b}{m}v = a = \frac{dv}{dt}$$

Separate Variables

$$\frac{dv}{g - \frac{b}{m}v} = dt$$

Integrate

$$\int_0^v \frac{dv}{g - \frac{b}{m}v} = \int_0^t dt$$

$$\frac{-m}{b}\left[\ln\left(g - \frac{b}{m}v\right) - \ln(g)\right] = t$$

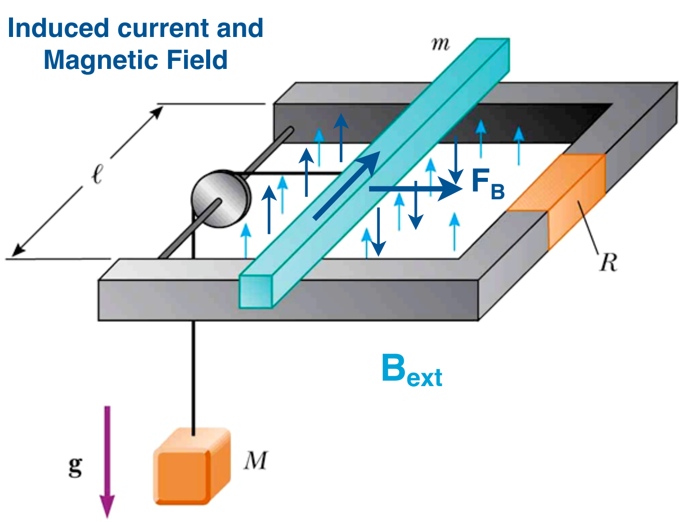
take exponential of both sides

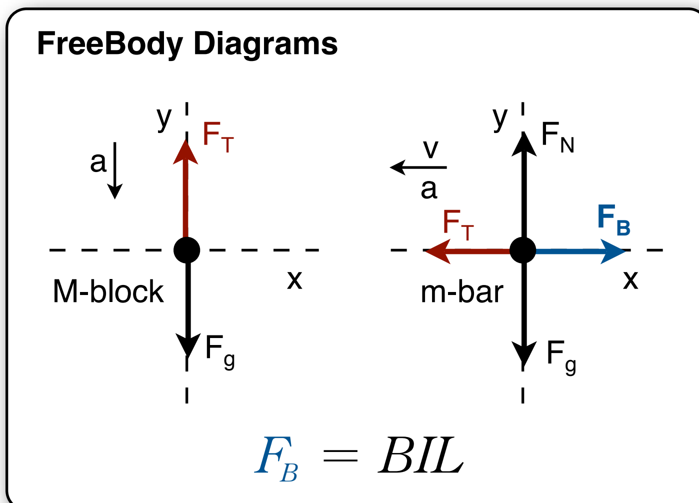
$$\left(g - \frac{b}{m}v\right)/g = e^{-bt/m}$$

$$v(t) = \frac{mg}{b}\left(1 - e^{-bt/m}\right)$$

Physics 196 Chapter 31 Problem 66

The **bar** of mass *m* in is pulled horizontally across parallel rails by a massless string that passes over an ideal pulley and is attached to a suspended **mass** ***M***. The uniform magnetic field has a magnitude *B*, and the distance between the rails is ℓ. The rails are connected at one end by a load resistor *R*. Derive an expression that gives the horizontal speed of the bar as a function of time.





Faraday's Law

$$\mathcal{E} = \frac{-d\Phi_B}{dt}$$

The *magnetic flux is changing* through the area bounded by the bar and the rails. According to **Faraday's Law** a *changing magnetic flux* will induce an **emf** in the circuit which will then produce a **current** which will then produce an **induced magnetic field** that will **oppose** the *changing magnetic flux*.

Looking at the diagram, the direction of the induced current is counterclockwise. This will produce a **magnetic field** that will add to the external magnetic field through the circuit. Since the arrows are pointing in the same direction there is a *repulsion* between the **induced** and the **external** magnetic fields to the right.

Since the magnetic flux is decreasing through the circuit the induced magnetic field will produce a force on the bar that will **oppose** this change.

Sum of the Forces

$$\Sigma F_y = F_T - F_g = -Ma_y \qquad \Sigma F_x = F_B - F_T = -ma_x$$

$$a_y = a_x$$

Falling Mass

$$F_T = mg - Ma$$

Sliding Bar

$$F_T = F_B + ma$$

We need to find the induced magnetic force

$$\Phi_B = BA\cos\theta$$

$$I_{ind} = \frac{\varepsilon}{R} = \frac{d\Phi_B}{dt}\frac{1}{R} = \frac{d(BA)}{dt}\frac{1}{R} = \frac{d(Blw)}{dt}\frac{1}{R} = Bl\frac{dw}{dt}\frac{1}{R} = \frac{Blv}{R}$$

$$F_B = BI_{ind}L = B\left(\frac{Blv}{R}\right)l = \frac{B^2l^2v}{R}$$

Solve for the acceleration

$$mg - Ma = F_B + ma$$

$$mg - F_B = Ma + ma$$

$$\frac{mg - F_B}{M+m} = a$$

$$\frac{mg - F_B}{M+m} = \frac{dv}{dt}$$

Now, we are set to solve for the velocity of the bar with respect to time

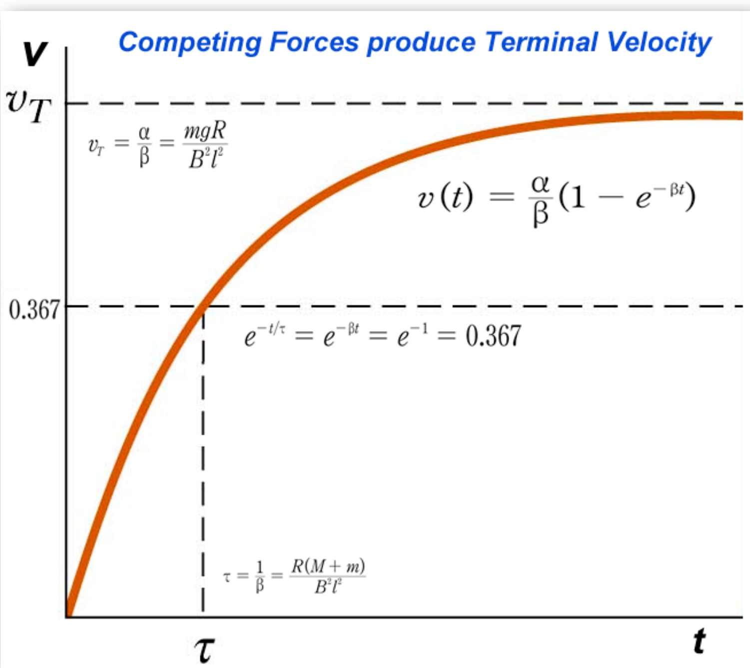
$$\frac{mg}{M+m} + \frac{B^2l^2v}{R(M+m)} = \frac{dv}{dt}$$

Let's reduce our stress and set

$$\alpha = \frac{mg}{M+m} \qquad \beta = \frac{B^2l^2}{R(M+m)}$$

Separate Variables

$$\frac{dv}{\frac{mg}{M+m} + \frac{B^2l^2v}{R(M+m)}} = dt$$



Integrate

$$\int_0^v \frac{dv}{\alpha - \beta v} = \int_0^t dt$$

$$\frac{-1}{\beta}[\ln(\alpha - \beta v) - \ln\alpha] = t$$

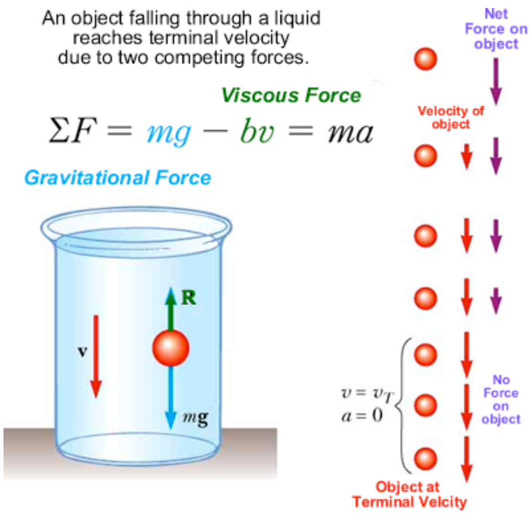
$$\ln(\alpha - \beta v) - \ln\alpha = -\beta t$$

$$(\alpha - \beta v)/\alpha = e^{-\beta t}$$

$$v(t) = \frac{\alpha}{\beta}(1 - e^{-\beta t})$$

Over time, this **magnetic force** will balance with the **gravitational force** on the bar. This **magnetic force** is dependent upon the velocity of the bar. The bar will reach a terminal velocity once these two forces acting on the bar are equal.

A Mechanical Analog



Sum of the Forces

$$\Sigma F = mg - bv = ma$$

Solve for the acceleration

$$g - \frac{b}{m}v = a = \frac{dv}{dt}$$

Separate Variables

$$\frac{dv}{g - \frac{b}{m}v} = dt$$

$$\int_0^v \frac{dv}{g - \frac{b}{m}v} = \int_0^t dt$$

Integrate

2 pts

$$\left(g - \frac{b}{m}v\right)/g = e^{-bt/m}$$

Solve for Velocity

2 pts

Physics 196 Chapter 31 Problem 66

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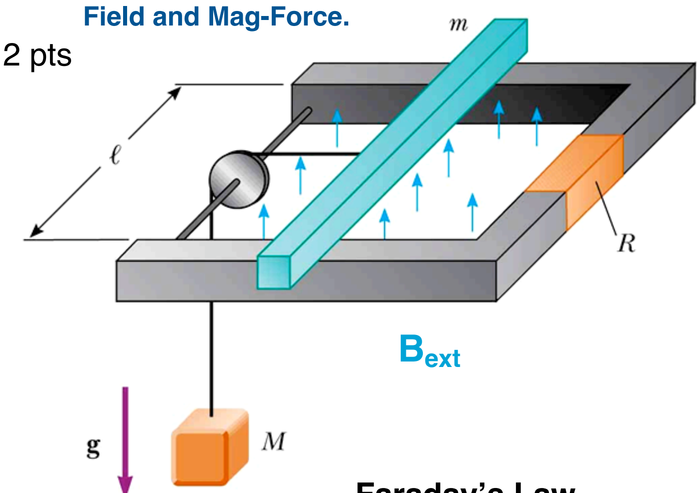
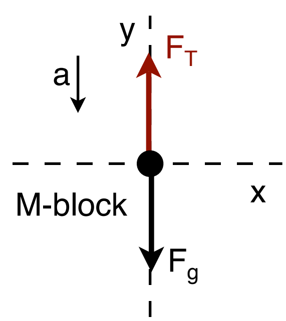
Draw Induced current, Magnetic Field and Mag-Force.

2 pts



FreeBody Diagrams



3 pts

Faraday's Law

1 pts

The *magnetic flux is changing* through the area bounded by the bar and the rails. According to **Faraday's Law** a *changing magnetic flux* will induce an **emf** in the circuit which will then produce a **current** which will then produce an **induced magnetic field** that will **oppose** the *changing magnetic flux*.

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Sum of the Forces

$$\Sigma F_y = F_T - F_g = -Ma_y$$

1 pts

Falling Mass

$$a_y = a_x$$

Sliding Bar

$$F_T = mg - Ma$$

1 pts

We need to find the induced magnetic force

$$\Phi_B = BA\cos\theta$$

$$I_{ind} = \frac{\varepsilon}{R} =$$

2 pts

$$F_B = BI_{ind}L =$$

2 pts

Set eq's equal and solve for the acceleration

2 pts

$$\frac{mg - F_B}{M+m} = a$$

$$\frac{mg - F_B}{M+m} = \frac{dv}{dt}$$

Plug in F_B and simplify.

2 pts

Let's reduce our stress and set

$$\alpha = \frac{mg}{M+m}$$

$$\beta = \frac{B^2 l^2}{R(M+m)}$$

Separate Variables

2 pts

$$\int_0^v \frac{dv}{\alpha - \beta v} = \int_0^t dt$$

Integrate

2 pts

$$(\alpha - \beta v)/\alpha = e^{-\beta t}$$

Solve for Velocity

2 pts

Draw the graph of the velocity wrt time. What is the terminal velocity? Show on the graph the velocity at e^{-2}?

4 pts