College Physics

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Rice University
6100 Main Street MS-380
Houston, Texas 77005

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PREFACE

About OpenStax College

OpenStax College is a non-profit organization committed to improving student access to quality learning materials. Our free textbooks are developed and peer-reviewed by educators to ensure they are readable, accurate, and meet the scope and sequence requirements of modern college courses. Unlike traditional textbooks, OpenStax College resources live online and are owned by the community of educators using them. Through our partnerships with companies and foundations committed to reducing costs for students, OpenStax College is working to improve access to higher education for all. OpenStax College is an initiative of Rice University and is made possible through the generous support of several philanthropic foundations.

About This Book

Welcome to *College Physics*, an OpenStax College resource created with several goals in mind: accessibility, affordability, customization, and student engagement—all while encouraging learners toward high levels of learning. Instructors and students alike will find that this textbook offers a strong foundation in introductory physics, with algebra as a prerequisite. It is available for free online and in low-cost print and e-book editions.

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To the Student

This book is written for you. It is based on the teaching and research experience of numerous physicists and influenced by a strong recollection of their own struggles as students. After reading this book, we hope you see that physics is visible everywhere. Applications range from driving a car to launching a rocket, from a skater whirling on ice to a neutron star spinning in space, and from taking your temperature to taking a chest X-ray.

To the Instructor

This text is intended for one-year introductory courses requiring algebra and some trigonometry, but no calculus. OpenStax College provides the essential supplemental resources at http://openstaxcollege.org ; however, we have pared down the number of supplements to keep costs low. College Physics can be easily customized for your course using Connexions (http://cnx.org/content/col11406). Simply select the content most relevant to your curriculum and create a textbook that speaks directly to the needs of your class.

General Approach

College Physics is organized such that topics are introduced conceptually with a steady progression to precise definitions and analytical applications. The analytical aspect (problem solving) is tied back to the conceptual before moving on to another topic. Each introductory chapter, for example, opens with an engaging photograph relevant to the subject of the chapter and interesting applications that are easy for most students to visualize.

Organization, Level, and Content

There is considerable latitude on the part of the instructor regarding the use, organization, level, and content of this book. By choosing the types of problems assigned, the instructor can determine the level of sophistication required of the student.

Concepts and Calculations

The ability to calculate does not guarantee conceptual understanding. In order to unify conceptual, analytical, and calculation skills within the learning process, we have integrated Strategies and Discussions throughout the text.

Modern Perspective

The chapters on modern physics are more complete than many other texts on the market, with an entire chapter devoted to medical applications of nuclear physics and another to particle physics. The final chapter of the text, “Frontiers of Physics,” is devoted to the most exciting endeavors in physics. It ends with a module titled “Some Questions We Know to Ask.”

Supplements

Accompanying the main text are a **Student Solutions Manual and an Instructor Solutions Manual (http://openstaxcollege.org/textbooks/college-physics)** . The Student Solutions Manual provides worked-out solutions to select end-of-module Problems and Exercises. The Instructor Solutions Manual provides worked-out solutions to all Exercises.

Features of OpenStax *College Physics*

The following briefly describes the special features of this text.

Modularity

This textbook is organized on Connexions (http://cnx.org) as a collection of modules that can be rearranged and modified to suit the needs of a particular professor or class. That being said, modules often contain references to content in other modules, as most topics in physics cannot be discussed in isolation.

Learning Objectives

Every module begins with a set of learning objectives. These objectives are designed to guide the instructor in deciding what content to include or assign, and to guide the student with respect to what he or she can expect to learn. After completing the module and end-of-module exercises, students should be able to demonstrate mastery of the learning objectives.

Call-Outs

Key definitions, concepts, and equations are called out with a special design treatment. Call-outs are designed to catch readers' attention, to make it clear that a specific term, concept, or equation is particularly important, and to provide easy reference for a student reviewing content.

Key Terms

Key terms are in bold and are followed by a definition in context. Definitions of key terms are also listed in the Glossary, which appears at the end of the module.

Worked Examples

Worked examples have four distinct parts to promote both analytical and conceptual skills. Worked examples are introduced in words, always using some application that should be of interest. This is followed by a Strategy section that emphasizes the concepts involved and how solving the problem relates to those concepts. This is followed by the mathematical Solution and Discussion.

Many worked examples contain multiple-part problems to help the students learn how to approach normal situations, in which problems tend to have multiple parts. Finally, worked examples employ the techniques of the problem-solving strategies so that students can see how those strategies succeed in practice as well as in theory.

Problem-Solving Strategies

Problem-solving strategies are first presented in a special section and subsequently appear at crucial points in the text where students can benefit most from them. Problem-solving strategies have a logical structure that is reinforced in the worked examples and supported in certain places by line drawings that illustrate various steps.

Misconception Alerts

Students come to physics with preconceptions from everyday experiences and from previous courses. Some of these preconceptions are misconceptions, and many are very common among students and the general public. Some are inadvertently picked up through misunderstandings of lectures and texts. The Misconception Alerts feature is designed to point these out and correct them explicitly.

Take-Home Investigations

Take Home Investigations provide the opportunity for students to apply or explore what they have learned with a hands-on activity.

Things Great and Small

In these special topic essays, macroscopic phenomena (such as air pressure) are explained with submicroscopic phenomena (such as atoms bouncing off walls). These essays support the modern perspective by describing aspects of modern physics before they are formally treated in later chapters. Connections are also made between apparently disparate phenomena.

Simulations

Where applicable, students are directed to the interactive PHeT physics simulations developed by the University of Colorado (**http://phet.colorado.edu (http://phet.colorado.edu)**). There they can further explore the physics concepts they have learned about in the module.

Summary

Module summaries are thorough and functional and present all important definitions and equations. Students are able to find the definitions of all terms and symbols as well as their physical relationships. The structure of the summary makes plain the fundamental principles of the module or collection and serves as a useful study guide.

Glossary

At the end of every module or chapter is a glossary containing definitions of all of the key terms in the module or chapter.

End-of-Module Problems

At the end of every chapter is a set of Conceptual Questions and/or skills-based Problems & Exercises. Conceptual Questions challenge students' ability to explain what they have learned conceptually, independent of the mathematical details. Problems & Exercises challenge students to apply both concepts and skills to solve mathematical physics problems. Online, every other problem includes an answer that students can reveal immediately by clicking on a “Show Solution” button. Fully worked solutions to select problems are available in the Student Solutions Manual and the Teacher Solutions Manual.

In addition to traditional skills-based problems, there are three special types of end-of-module problems: Integrated Concept Problems, Unreasonable Results Problems, and Construct Your Own Problems. All of these problems are indicated with a subtitle preceding the problem.

Integrated Concept Problems

In Unreasonable Results Problems, students are challenged not only to apply concepts and skills to solve a problem, but also to analyze the answer with respect to how likely or realistic it really is. These problems contain a premise that produces an unreasonable answer and are designed to further emphasize that properly applied physics must describe nature accurately and is not simply the process of solving equations.

Unreasonable Results

In Unreasonable Results Problems, students are challenged to not only apply concepts and skills to solve a problem, but also to analyze the answer with respect to how likely or realistic it really is. These problems contain a premise that produces an unreasonable answer and are designed to further emphasize that properly applied physics must describe nature accurately and is not simply the process of solving equations.

Construct Your Own Problem

These problems require students to construct the details of a problem, justify their starting assumptions, show specific steps in the problem's solution, and finally discuss the meaning of the result. These types of problems relate well to both conceptual and analytical aspects of physics, emphasizing that physics must describe nature. Often they involve an integration of topics from more than one chapter. Unlike other problems, solutions are not provided since there is no single correct answer. Instructors should feel free to direct students regarding the level and scope of their considerations. Whether the problem is solved and described correctly will depend on initial assumptions.

Appendices

Appendix A: Atomic Masses
Appendix B: Selected Radioactive Isotopes
Appendix C: Useful Information
Appendix D: Glossary of Key Symbols and Notation

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Senior Contributing Authors

Dr. Paul Peter Urone
Dr. Roger Hinrichs, State University of New York, College at Oswego

Contributing Authors

Dr. Kim Dirks, University of Auckland, New Zealand
Dr. Manjula Sharma, University of Sydney, Australia

Expert Reviewers

Erik Christensen, P.E, South Florida Community College
Dr. Eric Kincanon, Gonzaga University
Dr. Douglas Ingram, Texas Christian University
Lee H. LaRue, Paris Junior College
Dr. Marc Sher, College of William and Mary
Dr. Ulrich Zurcher, Cleveland State University
Dr. Matthew Adams, Crafton Hills College, San Bernardino Community College District
Dr. Chuck Pearson, Virginia Intermont College

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Webassign is an independent online homework and assessment system that has been available commercially since 1998. WebAssign has recently begun to support the Open Education Resource community by creating a high quality online homework solution for selected open-source textbooks, available at an affordable price to students. These question collections include randomized values and variables, immediate feedback, links to the open-source textbook, and a variety of text-specific resources and tools; as well as the same level of rigorous coding and accuracy-checking as any commercially available online homework solution supporting traditionally available textbooks.

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1 INTRODUCTION: THE NATURE OF SCIENCE AND PHYSICS

Figure 1.1 Galaxies are as immense as atoms are small. Yet the same laws of physics describe both, and all the rest of nature—an indication of the underlying unity in the universe. The laws of physics are surprisingly few in number, implying an underlying simplicity to nature's apparent complexity. (credit: NASA, JPL-Caltech, P. Barmby, Harvard-Smithsonian Center for Astrophysics)

Learning Objectives

1.1. Physics: An Introduction
- Explain the difference between a principle and a law.
- Explain the difference between a model and a theory.

1.2. Physical Quantities and Units
- Perform unit conversions both in the SI and English units.
- Explain the most common prefixes in the SI units and be able to write them in scientific notation.

1.3. Accuracy, Precision, and Significant Figures
- Determine the appropriate number of significant figures in both addition and subtraction, as well as multiplication and division calculations.
- Calculate the percent uncertainty of a measurement.

1.4. Approximation
- Make reasonable approximations based on given data.

Introduction to Science and the Realm of Physics, Physical Quantities, and Units

What is your first reaction when you hear the word "physics"? Did you imagine working through difficult equations or memorizing formulas that seem to have no real use in life outside the physics classroom? Many people come to the subject of physics with a bit of fear. But as you begin your exploration of this broad-ranging subject, you may soon come to realize that physics plays a much larger role in your life than you first thought, no matter your life goals or career choice.

For example, take a look at the image above. This image is of the Andromeda Galaxy, which contains billions of individual stars, huge clouds of gas, and dust. Two smaller galaxies are also visible as bright blue spots in the background. At a staggering 2.5 million light years from the Earth, this galaxy is the nearest one to our own galaxy (which is called the Milky Way). The stars and planets that make up Andromeda might seem to be the furthest thing from most people's regular, everyday lives. But Andromeda is a great starting point to think about the forces that hold together the universe. The forces that cause Andromeda to act as it does are the same forces we contend with here on Earth, whether we are planning to send a rocket into space or simply raise the walls for a new home. The same gravity that causes the stars of Andromeda to rotate and revolve also causes water to flow over hydroelectric dams here on Earth. Tonight, take a moment to look up at the stars. The forces out there are the same as the ones here on Earth. Through a study of physics, you may gain a greater understanding of the interconnectedness of everything we can see and know in this universe.

Think now about all of the technological devices that you use on a regular basis. Computers, smart phones, GPS systems, MP3 players, and satellite radio might come to mind. Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above tracks, "invisibility cloaks" that bend light around them, and microscopic robots that fight cancer cells in our bodies. All of these groundbreaking advancements, commonplace or unbelievable, rely on the principles of physics. Aside from playing a significant role in technology, professionals such as engineers, pilots, physicians, physical therapists, electricians, and computer programmers apply physics concepts in their daily work. For example, a pilot must understand how wind forces affect a flight path and a physical therapist must understand how the muscles in the body experience forces as they move and bend. As you will learn in this text, physics principles are propelling new, exciting technologies, and these principles are applied in a wide range of careers.

In this text, you will begin to explore the history of the formal study of physics, beginning with natural philosophy and the ancient Greeks, and leading up through a review of Sir Isaac Newton and the laws of physics that bear his name. You will also be introduced to the standards scientists use when they study physical quantities and the interrelated system of measurements most of the scientific community uses to communicate in a single mathematical language. Finally, you will study the limits of our ability to be accurate and precise, and the reasons scientists go to painstaking lengths to be as clear as possible regarding their own limitations.

1.1 Physics: An Introduction

Figure 1.2 The flight formations of migratory birds such as Canada geese are governed by the laws of physics. (credit: David Merrett)

The physical universe is enormously complex in its detail. Every day, each of us observes a great variety of objects and phenomena. Over the centuries, the curiosity of the human race has led us collectively to explore and catalog a tremendous wealth of information. From the flight of birds to the colors of flowers, from lightning to gravity, from quarks to clusters of galaxies, from the flow of time to the mystery of the creation of the universe, we have asked questions and assembled huge arrays of facts. In the face of all these details, we have discovered that a surprisingly small and unified set of physical laws can explain what we observe. As humans, we make generalizations and seek order. We have found that nature is remarkably cooperative—it exhibits the *underlying order and simplicity* we so value.

It is the underlying order of nature that makes science in general, and physics in particular, so enjoyable to study. For example, what do a bag of chips and a car battery have in common? Both contain energy that can be converted to other forms. The law of conservation of energy (which says that energy can change form but is never lost) ties together such topics as food calories, batteries, heat, light, and watch springs. Understanding this law makes it easier to learn about the various forms energy takes and how they relate to one another. Apparently unrelated topics are connected through broadly applicable physical laws, permitting an understanding beyond just the memorization of lists of facts.

The unifying aspect of physical laws and the basic simplicity of nature form the underlying themes of this text. In learning to apply these laws, you will, of course, study the most important topics in physics. More importantly, you will gain analytical abilities that will enable you to apply these laws far beyond the scope of what can be included in a single book. These analytical skills will help you to excel academically, and they will also help you to think critically in any professional career you choose to pursue. This module discusses the realm of physics (to define what physics is), some applications of physics (to illustrate its relevance to other disciplines), and more precisely what constitutes a physical law (to illuminate the importance of experimentation to theory).

Science and the Realm of Physics

Science consists of the theories and laws that are the general truths of nature as well as the body of knowledge they encompass. Scientists are continually trying to expand this body of knowledge and to perfect the expression of the laws that describe it. **Physics** is concerned with describing the interactions of energy, matter, space, and time, and it is especially interested in what fundamental mechanisms underlie every phenomenon. The concern for describing the basic phenomena in nature essentially defines the *realm of physics*.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smart phone (**Figure 1.3**). Physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS system. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics equations to determine the travel time from one location to another.

Figure 1.3 The Apple "iPhone" is a common smart phone with a GPS function. Physics describes the way that electricity flows through the circuits of this device. Engineers use their knowledge of physics to construct an iPhone with features that consumers will enjoy. One specific feature of an iPhone is the GPS function. GPS uses physics equations to determine the driving time between two locations on a map. (credit: @gletham GIS, Social, Mobile Tech Images)

Applications of Physics

You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. It can help you understand how microwave ovens work, why metals should not be put into them, and why they might affect pacemakers. (See **Figure 1.4** and **Figure 1.5**.) Physics allows you to understand the hazards of radiation and rationally evaluate these hazards more easily. Physics also explains the reason why a black car radiator helps remove heat in a car engine, and it explains why a white roof helps keep the inside of a house cool. Similarly, the operation of a car's ignition system as well as the transmission of electrical signals through our body's nervous system are much easier to understand when you think about them in terms of basic physics.

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry, for example—since it deals with the interactions of atoms and molecules—is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability, and is involved in the acoustics, heating, lighting, and cooling of buildings. Parts of geology rely heavily on physics, such as radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes (**Figure 1.6** and **Figure 1.7**). On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics; for example, cancer radiotherapy uses ionizing radiation. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

It is not necessary to formally study all applications of physics. What is most useful is knowledge of the basic laws of physics and a skill in the analytical methods for applying them. The study of physics also can improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences, and the study of physics makes other sciences easier to understand.

Figure 1.4 The laws of physics help us understand how common appliances work. For example, the laws of physics can help explain how microwave ovens heat up food, and they also help us understand why it is dangerous to place metal objects in a microwave oven. (credit: MoneyBlogNewz)

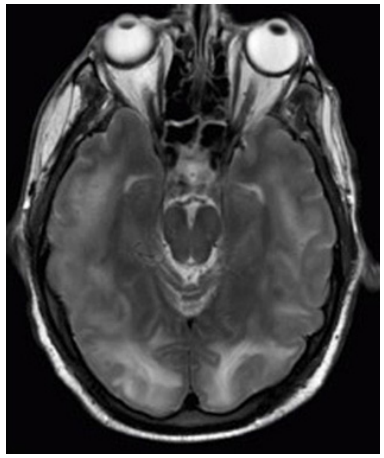


Figure 1.5 These two applications of physics have more in common than meets the eye. Microwave ovens use electromagnetic waves to heat food. Magnetic resonance imaging (MRI) also uses electromagnetic waves to yield an image of the brain, from which the exact location of tumors can be determined. (credit: Rashmi Chawla, Daniel Smith, and Paul E. Marik)

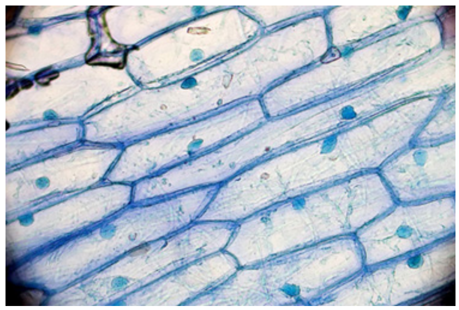


Figure 1.6 Physics, chemistry, and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. (credit: Umberto Salvagnin)

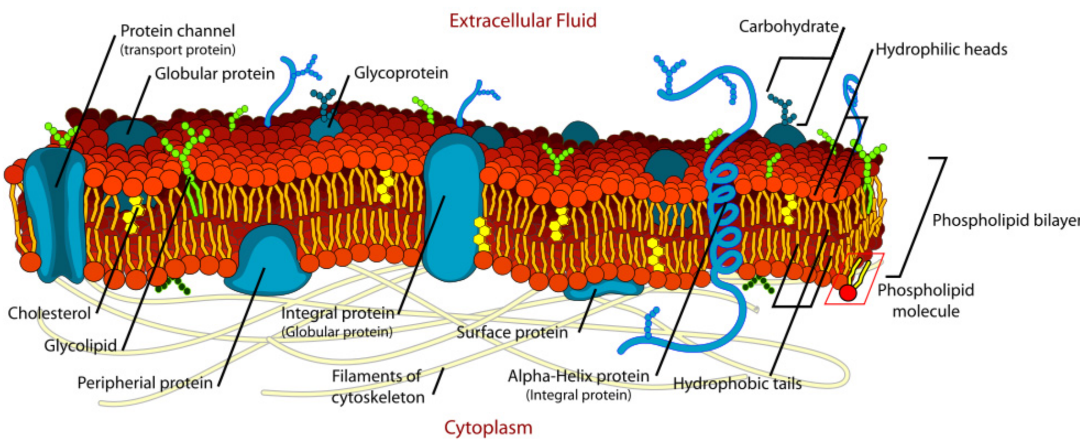


Figure 1.7 An artist's rendition of the the structure of a cell membrane. Membranes form the boundaries of animal cells and are complex in structure and function. Many of the most fundamental properties of life, such as the firing of nerve cells, are related to membranes. The disciplines of biology, chemistry, and physics all help us understand the membranes of animal cells. (credit: Mariana Ruiz)

Models, Theories, and Laws; The Role of Experimentation

The laws of nature are concise descriptions of the universe around us; they are human statements of the underlying laws or rules that all natural processes follow. Such laws are intrinsic to the universe; humans did not create them and so cannot change them. We can only discover and understand them. Their discovery is a very human endeavor, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. (See **Figure 1.8** and **Figure 1.9**.) The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be.

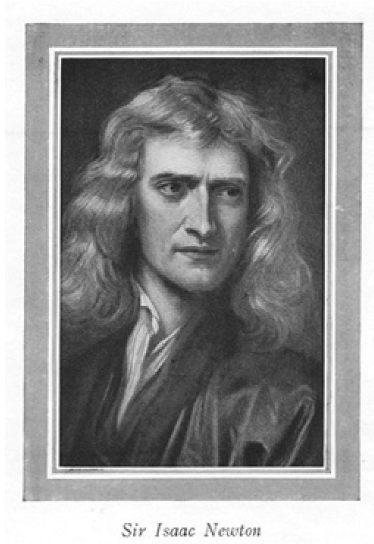


Figure 1.8 **Isaac Newton** (1642–1727) was very reluctant to publish his revolutionary work and had to be convinced to do so. In his later years, he stepped down from his academic post and became exchequer of the Royal Mint. He took this post seriously, inventing reeding (or creating ridges) on the edge of coins to prevent unscrupulous people from trimming the silver off of them before using them as currency. (credit: Arthur Shuster and Arthur E. Shipley: *Britain's Heritage of Science*. London, 1917.)

Figure 1.9 **Marie Curie** (1867–1934) sacrificed monetary assets to help finance her early research and damaged her physical well-being with radiation exposure. She is the only person to win Nobel prizes in both physics and chemistry. One of her daughters also won a Nobel Prize. (credit: Wikimedia Commons)

We all are curious to some extent. We look around, make generalizations, and try to understand what we see—for example, we look up and wonder whether one type of cloud signals an oncoming storm. As we become serious about exploring nature, we become more organized and formal in collecting and analyzing data. We attempt greater precision, perform controlled experiments (if we can), and write down ideas about how the data may be organized and unified. We then formulate models, theories, and laws based on the data we have collected and analyzed to generalize and communicate the results of these experiments.

A **model** is a representation of something that is often too difficult (or impossible) to display directly. While a model is justified with experimental proof, it is only accurate under limited situations. An example is the planetary model of the atom in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun. (See **Figure 1.10**.) We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases (atomic spectra). Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation. A **theory** is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. Some theories include models to help visualize phenomena, whereas others do not. Newton's theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, is a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

A **law** uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation *law* is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton's second law of motion, which relates force, mass, and acceleration by the simple equation $\mathbf{F} = m\mathbf{a}$. A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a theory is much more complex and dynamic. A law describes a single action, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Less broadly applicable statements are usually called principles (such as Pascal's principle, which is applicable only in fluids), but the distinction between laws and principles often is not carefully made.

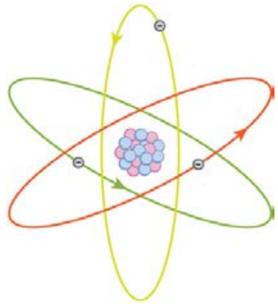


Figure 1.10 What is a model? This planetary model of the atom shows electrons orbiting the nucleus. It is a drawing that we use to form a mental image of the atom that we cannot see directly with our eyes because it is too small.

Models, Theories, and Laws

Models, theories, and laws are used to help scientists analyze the data they have already collected. However, often after a model, theory, or law has been developed, it points scientists toward new discoveries they would not otherwise have made.

The models, theories, and laws we devise sometimes *imply the existence of objects or phenomena as yet unobserved.* These predictions are remarkable triumphs and tributes to the power of science. It is the underlying order in the universe that enables scientists to make such spectacular predictions. However, if *experiment* does not verify our predictions, then the theory or law is wrong, no matter how elegant or convenient it is. Laws can never be known with absolute certainty because it is impossible to perform every imaginable experiment in order to confirm a law in every possible scenario. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

The study of science in general and physics in particular is an adventure much like the exploration of uncharted ocean. Discoveries are made; models, theories, and laws are formulated; and the beauty of the physical universe is made more sublime for the insights gained.

The Scientific Method

As scientists inquire and gather information about the world, they follow a process called the **scientific method**. This process typically begins with an observation and question that the scientist will research. Next, the scientist typically performs some research about the topic and then devises a hypothesis. Then, the scientist will test the hypothesis by performing an experiment. Finally, the scientist analyzes the results of the experiment and draws a conclusion. Note that the scientific method can be applied to many situations that are not limited to science, and this method can be modified to suit the situation.

Consider an example. Let us say that you try to turn on your car, but it will not start. You undoubtedly wonder: Why will the car not start? You can follow a scientific method to answer this question. First off, you may perform some research to determine a variety of reasons why the car will not start. Next, you will state a hypothesis. For example, you may believe that the car is not starting because it has no engine oil. To test this, you open the hood of the car and examine the oil level. You observe that the oil is at an acceptable level, and you thus conclude that the oil level is not contributing to your car issue. To troubleshoot the issue further, you may devise a new hypothesis to test and then repeat the process again.

The Evolution of Natural Philosophy into Modern Physics

Physics was not always a separate and distinct discipline. It remains connected to other sciences to this day. The word *physics* comes from Greek, meaning nature. The study of nature came to be called “natural philosophy.” From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, physics, mathematics, and medicine. Over the last few centuries, the growth of knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. (See **Figure 1.11**, **Figure 1.12**, and **Figure 1.13**.) Physics as it developed from the Renaissance to the end of the 19th century is called **classical physics**. It was transformed into modern physics by revolutionary discoveries made starting at the beginning of the 20th century.

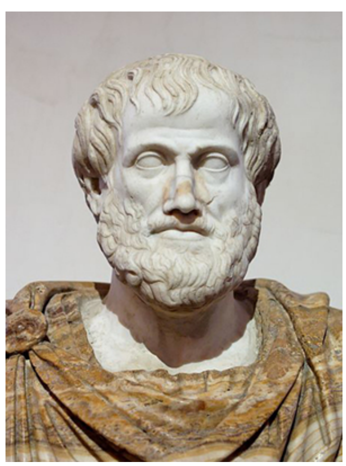


Figure 1.11 Over the centuries, natural philosophy has evolved into more specialized disciplines, as illustrated by the contributions of some of the greatest minds in history. The Greek philosopher **Aristotle** (384–322 B.C.) wrote on a broad range of topics including physics, animals, the soul, politics, and poetry. (credit: Jastrow (2006)/Ludovisi Collection)

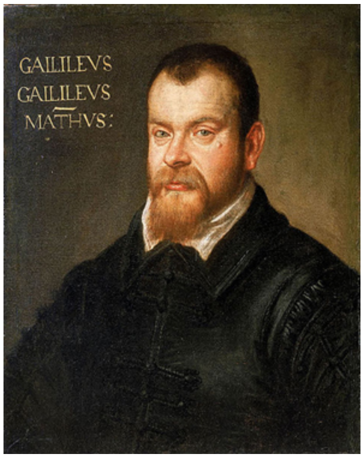


Figure 1.12 **Galileo Galilei** (1564–1642) laid the foundation of modern experimentation and made contributions in mathematics, physics, and astronomy. (credit: Domenico Tintoretto)

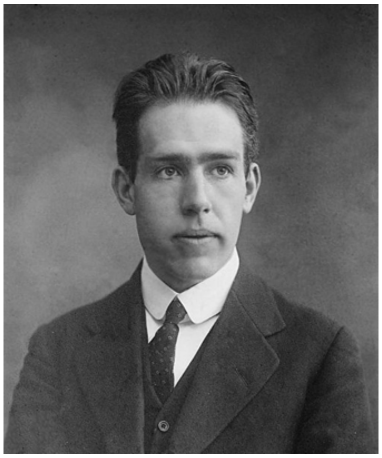


Figure 1.13 **Niels Bohr** (1885–1962) made fundamental contributions to the development of quantum mechanics, one part of modern physics. (credit: United States Library of Congress Prints and Photographs Division)

Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields, such as the field generated by the Earth, can be involved. Because humans live under such circumstances, classical physics seems intuitively reasonable, while many aspects of modern physics seem bizarre. This is why models are so useful in modern physics—they let us conceptualize phenomena we do not ordinarily experience. We can relate to models in human terms and visualize what happens when objects move at high speeds or imagine what objects too small to observe with our senses might be like. For example, we can understand an atom's properties because we can picture it in our minds, although we have never seen an atom with our eyes. New tools, of course, allow us to better picture phenomena we cannot see. In fact, new instrumentation has allowed us in recent years to actually "picture" the atom.

Limits on the Laws of Classical Physics

For the laws of classical physics to apply, the following criteria must be met: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields (such as the field generated by the Earth) can be involved.

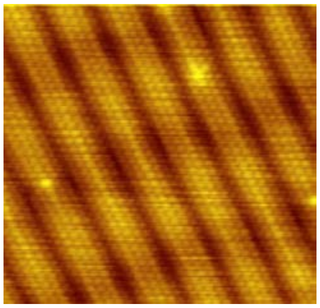


Figure 1.14 Using a scanning tunneling microscope (STM), scientists can see the individual atoms that compose this sheet of gold. (credit: Erwinrossen)

Some of the most spectacular advances in science have been made in modern physics. Many of the laws of classical physics have been modified or rejected, and revolutionary changes in technology, society, and our view of the universe have resulted. Like science fiction, modern physics is filled with fascinating objects beyond our normal experiences, but it has the advantage over science fiction of being very real. Why, then, is the majority of this text devoted to topics of classical physics? There are two main reasons: Classical physics gives an extremely accurate description of the universe under a wide range of everyday circumstances, and knowledge of classical physics is necessary to understand modern physics.

Modern physics itself consists of the two revolutionary theories, relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. **Relativity** must be used whenever an object is traveling at greater than about 1% of the speed of light or experiences a strong gravitational field such as that near the Sun. **Quantum mechanics** must be used for objects smaller than can be seen with a microscope. The combination of these two theories is *relativistic quantum mechanics,* and it describes the behavior of small objects traveling at high speeds or experiencing a strong gravitational field. Relativistic quantum mechanics is the best universally applicable theory we have. Because of its mathematical complexity, it is used only when necessary, and the other theories are used whenever they will produce sufficiently accurate results. We will find, however, that we can do a great deal of modern physics with the algebra and trigonometry used in this text.

Check Your Understanding

A friend tells you he has learned about a new law of nature. What can you know about the information even before your friend describes the law? How would the information be different if your friend told you he had learned about a scientific theory rather than a law?

Solution

Without knowing the details of the law, you can still infer that the information your friend has learned conforms to the requirements of all laws of nature: it will be a concise description of the universe around us; a statement of the underlying rules that all natural processes follow. If the information had been a theory, you would be able to infer that the information will be a large-scale, broadly applicable generalization.

PhET Explorations: Equation Grapher

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = bx$) to see how they add to generate the polynomial curve.

PhET Interactive Simulation

Figure 1.15 Equation Grapher (http://cnx.org/content/m42092/1.4/equation-grapher_en.jar)

1.2 Physical Quantities and Units

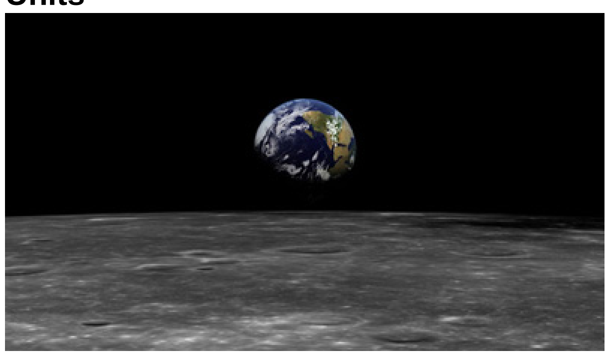


Figure 1.16 The distance from Earth to the Moon may seem immense, but it is just a tiny fraction of the distances from Earth to other celestial bodies. (credit: NASA)

The range of objects and phenomena studied in physics is immense. From the incredibly short lifetime of a nucleus to the age of the Earth, from the tiny sizes of sub-nuclear particles to the vast distance to the edges of the known universe, from the force exerted by a jumping flea to the force between Earth and the Sun, there are enough factors of 10 to challenge the imagination of even the most experienced scientist. Giving numerical values for physical quantities and equations for physical principles allows us to understand nature much more deeply than does qualitative description alone. To comprehend these vast ranges, we must also have accepted units in which to express them. And we shall find that (even in the potentially mundane discussion of meters, kilograms, and seconds) a profound simplicity of nature appears—all physical quantities can be expressed as combinations of only four fundamental physical quantities: length, mass, time, and electric current.

We define a **physical quantity** either by *specifying how it is measured* or by *stating how it is calculated* from other measurements. For example, we define distance and time by specifying methods for measuring them, whereas we define *average speed* by stating that it is calculated as distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of **units**, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way. (See **Figure 1.17**.)

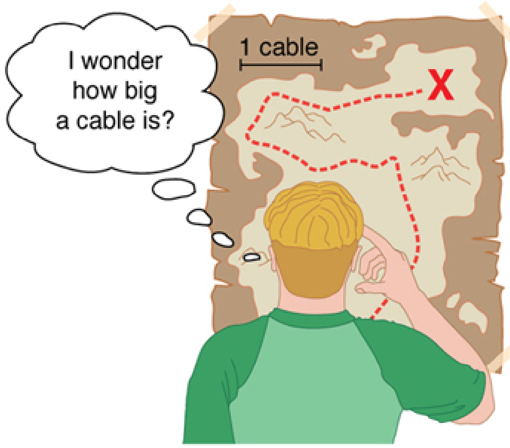


Figure 1.17 Distances given in unknown units are maddeningly useless.

There are two major systems of units used in the world: **SI units** (also known as the metric system) and **English units** (also known as the customary or imperial system). **English units** were historically used in nations once ruled by the British Empire and are still widely used in the United States. Virtually every other country in the world now uses SI units as the standard; the metric system is also the standard system agreed upon by scientists and mathematicians. The acronym "SI" is derived from the French *Système International*.

SI Units: Fundamental and Derived Units

Table 1.1 gives the fundamental SI units that are used throughout this textbook. This text uses non-SI units in a few applications where they are in very common use, such as the measurement of blood pressure in millimeters of mercury (mm Hg). Whenever non-SI units are discussed, they will be tied to SI units through conversions.

Table 1.1 Fundamental SI Units

Length	Mass	Time	Electric Current
meter (m)	kilogram (kg)	second (s)	ampere (A)

It is an intriguing fact that some physical quantities are more fundamental than others and that the most fundamental physical quantities can be defined *only* in terms of the procedure used to measure them. The units in which they are measured are thus called **fundamental units**. In this textbook, the fundamental physical quantities are taken to be length, mass, time, and electric current. (Note that electric current will not be introduced until much later in this text.) All other physical quantities, such as force and electric charge, can be expressed as algebraic combinations of length, mass, time, and current (for example, speed is length divided by time); these units are called **derived units**.

Units of Time, Length, and Mass: The Second, Meter, and Kilogram

The Second

The SI unit for time, the **second**(abbreviated s), has a long history. For many years it was defined as 1/86,400 of a mean solar day. More recently, a new standard was adopted to gain greater accuracy and to define the second in terms of a non-varying, or constant, physical phenomenon (because the solar day is getting longer due to very gradual slowing of the Earth's rotation). Cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. In 1967 the second was redefined as the time required for 9,192,631,770 of these vibrations. (See **Figure 1.18**.) Accuracy in the fundamental units is essential, because all measurements are ultimately expressed in terms of fundamental units and can be no more accurate than are the fundamental units themselves.

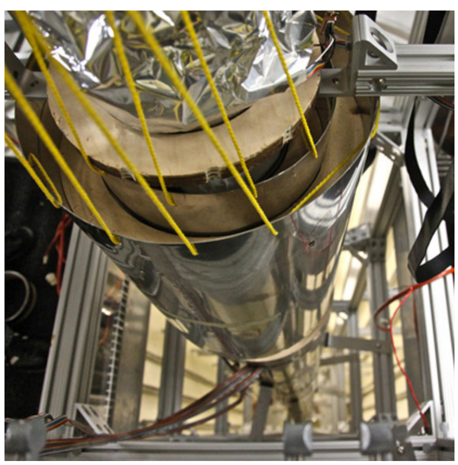


Figure 1.18 An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of better than a microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic fountain nearly 30 feet tall! (credit: Steve Jurvetson/Flickr)

The Meter

The SI unit for length is the **meter** (abbreviated m); its definition has also changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar now kept near Paris. By 1960, it had become possible to define the meter even more accurately in terms of the wavelength of light, so it was again redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition (partly for greater accuracy) as the distance light travels in a vacuum in 1/299,792,458 of a second. (See **Figure 1.19**.) This change defines the speed of light to be exactly 299,792,458 meters per second. The length of the meter will change if the speed of light is someday measured with greater accuracy.

The Kilogram

The SI unit for mass is the **kilogram** (abbreviated kg); it is defined to be the mass of a platinum-iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. Exact replicas of the standard kilogram are also kept at the United States' National Institute of Standards and Technology, or NIST, located in Gaithersburg, Maryland outside of Washington D.C., and at other locations around the world. The determination of all other masses can be ultimately traced to a comparison with the standard mass.

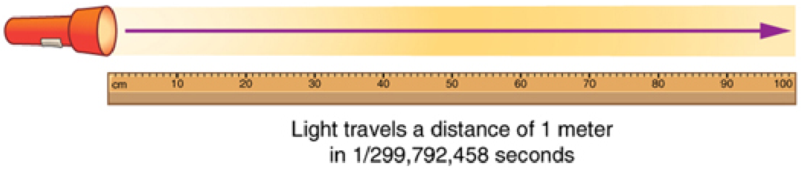


Figure 1.19 The meter is defined to be the distance light travels in 1/299,792,458 of a second in a vacuum. Distance traveled is speed multiplied by time.

Electric current and its accompanying unit, the ampere, will be introduced in **Introduction to Electric Current, Resistance, and Ohm's Law** when electricity and magnetism are covered. The initial modules in this textbook are concerned with mechanics, fluids, heat, and waves. In these subjects all pertinent physical quantities can be expressed in terms of the fundamental units of length, mass, and time.

Metric Prefixes

SI units are part of the **metric system**. The metric system is convenient for scientific and engineering calculations because the units are categorized by factors of 10. **Table 1.2** gives metric prefixes and symbols used to denote various factors of 10.

Metric systems have the advantage that conversions of units involve only powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as the system of U.S. customary units, the relationships are not as simple—there are 12 inches in a foot, 5280 feet in a mile, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by using an appropriate metric prefix. For example, distances in meters are suitable in construction, while distances in kilometers are appropriate for air travel, and the tiny measure of nanometers are convenient in optical design. With the metric system there is no need to invent new units for particular applications.

The term **order of magnitude** refers to the scale of a value expressed in the metric system. Each power of 10 in the metric system represents a different order of magnitude. For example, 10^1, 10^2, 10^3, and so forth are all different orders of magnitude. All quantities that can be expressed as a product of a specific power of 10 are said to be of the *same* order of magnitude. For example, the number 800 can be written as 8×10^2, and the number 450 can be written as 4.5×10^2. Thus, the numbers 800 and 450 are of the same order of magnitude: 10^2. Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of $10^{-9}\text{ m},$ while the diameter of the Sun is on the order of $10^9\text{ m}.$

The Quest for Microscopic Standards for Basic Units

The fundamental units described in this chapter are those that produce the greatest accuracy and precision in measurement. There is a sense among physicists that, because there is an underlying microscopic substructure to matter, it would be most satisfying to base our standards of measurement on microscopic objects and fundamental physical phenomena such as the speed of light. A microscopic standard has been accomplished for the standard of time, which is based on the oscillations of the cesium atom.

The standard for length was once based on the wavelength of light (a small-scale length) emitted by a certain type of atom, but it has been supplanted by the more precise measurement of the speed of light. If it becomes possible to measure the mass of atoms or a particular arrangement of atoms such as a silicon sphere to greater precision than the kilogram standard, it may become possible to base mass measurements on the small scale. There are also possibilities that electrical phenomena on the small scale may someday allow us to base a unit of charge on the charge of electrons and protons, but at present current and charge are related to large-scale currents and forces between wires.

Table 1.2 Metric Prefixes for Powers of 10 and their Symbols

Prefix	Symbol	Value[1]	Example (some are approximate)			
exa	E	10^{18}	exameter	Em	10^{18} m	distance light travels in a century
peta	P	10^{15}	petasecond	Ps	10^{15} s	30 million years
tera	T	10^{12}	terawatt	TW	10^{12} W	powerful laser output
giga	G	10^{9}	gigahertz	GHz	10^{9} Hz	a microwave frequency
mega	M	10^{6}	megacurie	MCi	10^{6} Ci	high radioactivity
kilo	k	10^{3}	kilometer	km	10^{3} m	about 6/10 mile
hecto	h	10^{2}	hectoliter	hL	10^{2} L	26 gallons
deka	da	10^{1}	dekagram	dag	10^{1} g	teaspoon of butter
—	—	10^{0} (=1)				
deci	d	10^{-1}	deciliter	dL	10^{-1} L	less than half a soda
centi	c	10^{-2}	centimeter	cm	10^{-2} m	fingertip thickness
milli	m	10^{-3}	millimeter	mm	10^{-3} m	flea at its shoulders
micro	µ	10^{-6}	micrometer	µm	10^{-6} m	detail in microscope
nano	n	10^{-9}	nanogram	ng	10^{-9} g	small speck of dust
pico	p	10^{-12}	picofarad	pF	10^{-12} F	small capacitor in radio
femto	f	10^{-15}	femtometer	fm	10^{-15} m	size of a proton
atto	a	10^{-18}	attosecond	as	10^{-18} s	time light crosses an atom

Known Ranges of Length, Mass, and Time

The vastness of the universe and the breadth over which physics applies are illustrated by the wide range of examples of known lengths, masses, and times in **Table 1.3**. Examination of this table will give you some feeling for the range of possible topics and numerical values. (See **Figure 1.20** and **Figure 1.21**.)

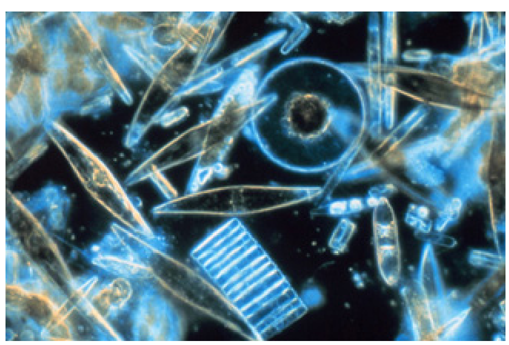


Figure 1.20 Tiny phytoplankton swims among crystals of ice in the Antarctic Sea. They range from a few micrometers to as much as 2 millimeters in length. (credit: Prof. Gordon T. Taylor, Stony Brook University; NOAA Corps Collections)

1. See **Appendix A** for a discussion of powers of 10.

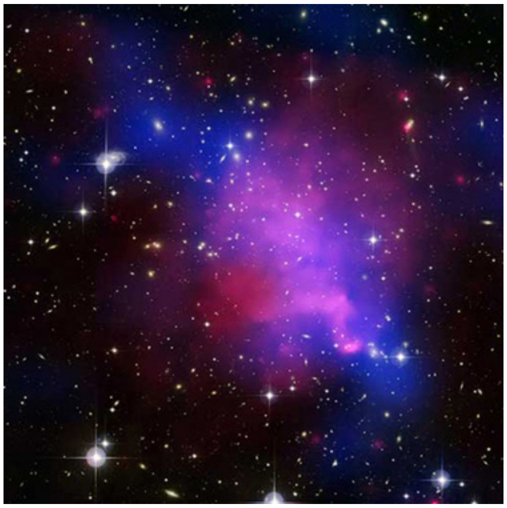


Figure 1.21 Galaxies collide 2.4 billion light years away from Earth. The tremendous range of observable phenomena in nature challenges the imagination. (credit: NASA/CXC/UVic./A. Mahdavi et al. Optical/lensing: CFHT/UVic./H. Hoekstra et al.)

Unit Conversion and Dimensional Analysis

It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters and you need to convert them to cups. Or, perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you will need to convert units of feet to miles.

Let us consider a simple example of how to convert units. Let us say that we want to convert 80 meters (m) to kilometers (km).

The first thing to do is to list the units that you have and the units that you want to convert to. In this case, we have units in *meters* and we want to convert to *kilometers*.

Next, we need to determine a **conversion factor** relating meters to kilometers. A conversion factor is a ratio expressing how many of one unit are equal to another unit. For example, there are 12 inches in 1 foot, 100 centimeters in 1 meter, 60 seconds in 1 minute, and so on. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor so that the units cancel out, as shown:

$$80\,\cancel{\text{m}} \times \frac{1\text{ km}}{1000\,\cancel{\text{m}}} = 0.080\text{ km}. \tag{1.1}$$

Note that the unwanted m unit cancels, leaving only the desired km unit. You can use this method to convert between any types of unit.

Click **Appendix C** for a more complete list of conversion factors.

Table 1.3 Approximate Values of Length, Mass, and Time

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
10^{-18}	Present experimental limit to smallest observable detail	10^{-30}	Mass of an electron $\left(9.11\times10^{-31}\text{ kg}\right)$	10^{-23}	Time for light to cross a proton
10^{-15}	Diameter of a proton	10^{-27}	Mass of a hydrogen atom $\left(1.67\times10^{-27}\text{ kg}\right)$	10^{-22}	Mean life of an extremely unstable nucleus
10^{-14}	Diameter of a uranium nucleus	10^{-15}	Mass of a bacterium	10^{-15}	Time for one oscillation of visible light
10^{-10}	Diameter of a hydrogen atom	10^{-5}	Mass of a mosquito	10^{-13}	Time for one vibration of an atom in a solid
10^{-8}	Thickness of membranes in cells of living organisms	10^{-2}	Mass of a hummingbird	10^{-8}	Time for one oscillation of an FM radio wave
10^{-6}	Wavelength of visible light	1	Mass of a liter of water (about a quart)	10^{-3}	Duration of a nerve impulse
10^{-3}	Size of a grain of sand	10^{2}	Mass of a person	1	Time for one heartbeat
1	Height of a 4-year-old child	10^{3}	Mass of a car	10^{5}	One day $\left(8.64\times10^{4}\text{ s}\right)$
10^{2}	Length of a football field	10^{8}	Mass of a large ship	10^{7}	One year (y) $\left(3.16\times10^{7}\text{ s}\right)$
10^{4}	Greatest ocean depth	10^{12}	Mass of a large iceberg	10^{9}	About half the life expectancy of a human
10^{7}	Diameter of the Earth	10^{15}	Mass of the nucleus of a comet	10^{11}	Recorded history
10^{11}	Distance from the Earth to the Sun	10^{23}	Mass of the Moon $\left(7.35\times10^{22}\text{ kg}\right)$	10^{17}	Age of the Earth
10^{16}	Distance traveled by light in 1 year (a light year)	10^{25}	Mass of the Earth $\left(5.97\times10^{24}\text{ kg}\right)$	10^{18}	Age of the universe
10^{21}	Diameter of the Milky Way galaxy	10^{30}	Mass of the Sun $\left(1.99\times10^{30}\text{ kg}\right)$		
10^{22}	Distance from the Earth to the nearest large galaxy (Andromeda)	10^{42}	Mass of the Milky Way galaxy (current upper limit)		
10^{26}	Distance from the Earth to the edges of the known universe	10^{53}	Mass of the known universe (current upper limit)		

Example 1.1 Unit Conversions: A Short Drive Home

Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note: Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

Solution for (a)

(1) Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

$$\text{average speed} = \frac{\text{distance}}{\text{time}}. \tag{1.2}$$

(2) Substitute the given values for distance and time.

$$\text{average speed} = \frac{10.0\text{ km}}{20.0\text{ min}} = 0.500\frac{\text{km}}{\text{min}}. \tag{1.3}$$

(3) Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is 60 min/hr. Thus,

$$\text{average speed} = 0.500\frac{\text{km}}{\text{min}}\times\frac{60\text{ min}}{1\text{ h}} = 30.0\frac{\text{km}}{\text{h}}. \tag{1.4}$$

Discussion for (a)

To check your answer, consider the following:

(1) Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows:

$$\frac{\text{km}}{\text{min}}\times\frac{1\ \text{hr}}{60\ \text{min}}=\frac{1}{60}\frac{\text{km}\cdot\text{hr}}{\text{min}^2}, \tag{1.5}$$

which are obviously not the desired units of km/h.

(2) Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.

(3) Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/hr does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 minutes, so the precision of the conversion factor is perfect.

(4) Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

Solution for (b)

There are several ways to convert the average speed into meters per second.

(1) Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.

(2) Multiplying by these yields

$$\text{Average speed}=30.0\frac{\text{km}}{\text{h}}\times\frac{1\ \text{h}}{3{,}600\ \text{s}}\times\frac{1{,}000\ \text{m}}{1\ \text{km}}, \tag{1.6}$$

$$\text{Average speed}=8.33\frac{\text{m}}{\text{s}}. \tag{1.7}$$

Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

You may have noted that the answers in the worked example just covered were given to three digits. Why? When do you need to be concerned about the number of digits in something you calculate? Why not write down all the digits your calculator produces? The module **Accuracy, Precision, and Significant Figures** will help you answer these questions.

Nonstandard Units

While there are numerous types of units that we are all familiar with, there are others that are much more obscure. For example, a **firkin** is a unit of volume that was once used to measure beer. One firkin equals about 34 liters. To learn more about nonstandard units, use a dictionary or encyclopedia to research different "weights and measures." Take note of any unusual units, such as a barleycorn, that are not listed in the text. Think about how the unit is defined and state its relationship to SI units.

Check Your Understanding

Some hummingbirds beat their wings more than 50 times per second. A scientist is measuring the time it takes for a hummingbird to beat its wings once. Which fundamental unit should the scientist use to describe the measurement? Which factor of 10 is the scientist likely to use to describe the motion precisely? Identify the metric prefix that corresponds to this factor of 10.

Solution
The scientist will measure the time between each movement using the fundamental unit of seconds. Because the wings beat so fast, the scientist will probably need to measure in milliseconds, or 10^{-3} seconds. (50 beats per second corresponds to 20 milliseconds per beat.)

Check Your Understanding

One cubic centimeter is equal to one milliliter. What does this tell you about the different units in the SI metric system?

Solution
The fundamental unit of length (meter) is probably used to create the derived unit of volume (liter). The measure of a milliliter is dependent on the measure of a centimeter.

1.3 Accuracy, Precision, and Significant Figures

Figure 1.22 A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The “known masses” are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams. (credit: Serge Melki)

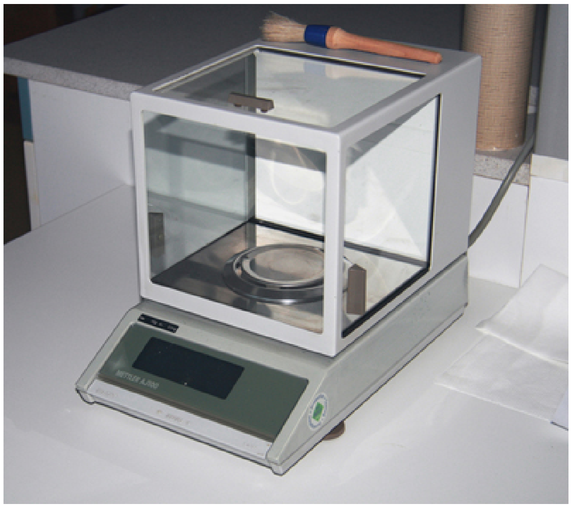


Figure 1.23 Many mechanical balances, such as double-pan balances, have been replaced by digital scales, which can typically measure the mass of an object more precisely. Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, many digital scales can measure the mass of an object up to the nearest thousandth of a gram. (credit: Karel Jakubec)

Accuracy and Precision of a Measurement

Science is based on observation and experiment—that is, on measurements. **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard computer paper. The packaging in which you purchased the paper states that it is 11.0 inches long. You measure the length of the paper three times and obtain the following measurements: 11.1 in., 11.2 in., and 10.9 in. These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate.

The **precision** of a measurement system is refers to how close the agreement is between repeated measurements (which are repeated under the same conditions). Consider the example of the paper measurements. The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements would be to determine the range, or difference, between the lowest and the highest measured values. In that case, the lowest value was 10.9 in. and the highest value was 11.2 in. Thus, the measured values deviated from each other by at most 0.3 in. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 10.9, 11.1, and 11.9, then the measurements would not be very precise because there would be significant variation from one measurement to another.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider an example of a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull’s-eye target, and think of each GPS attempt to locate the restaurant as a black dot. In **Figure 1.24**, you can see that the GPS measurements are spread out far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low precision, high accuracy measuring system. However, in **Figure 1.25**, the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system.

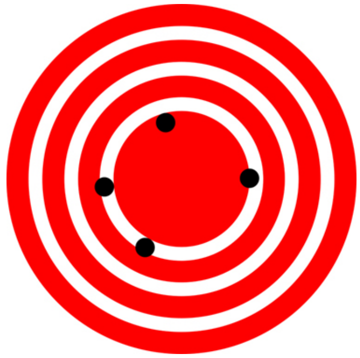


Figure 1.24 A GPS system attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (credit: Dark Evil)

Figure 1.25 In this figure, the dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (credit: Dark Evil)

Accuracy, Precision, and Uncertainty

The degree of accuracy and precision of a measuring system are related to the **uncertainty** in the measurements. Uncertainty is a quantitative measure of how much your measured values deviate from a standard or expected value. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45,000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44,500 miles or as high as 45,500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 in., plus or minus 0.2 in. The uncertainty in a measurement, A, is often denoted as δA ("delta A"), so the measurement result would be recorded as $A \pm \delta A$. In our paper example, the length of the paper could be expressed as $11 \text{ in.} \pm 0.2.$

The factors contributing to uncertainty in a measurement include:

1. Limitations of the measuring device,
2. The skill of the person making the measurement,
3. Irregularities in the object being measured,
4. Any other factors that affect the outcome (highly dependent on the situation).

In our example, such factors contributing to the uncertainty could be the following: the smallest division on the ruler is 0.1 in., the person using the ruler has bad eyesight, or one side of the paper is slightly longer than the other. At any rate, the uncertainty in a measurement must be based on a careful consideration of all the factors that might contribute and their possible effects.

Making Connections: Real-World Connections – Fevers or Chills?

Uncertainty is a critical piece of information, both in physics and in many other real-world applications. Imagine you are caring for a sick child. You suspect the child has a fever, so you check his or her temperature with a thermometer. What if the uncertainty of the thermometer were 3.0°C? If the child's temperature reading was 37.0°C (which is normal body temperature), the "true" temperature could be anywhere from a hypothermic 34.0°C to a dangerously high 40.0°C. A thermometer with an uncertainty of 3.0°C would be useless.

Percent Uncertainty

One method of expressing uncertainty is as a percent of the measured value. If a measurement A is expressed with uncertainty, δA, the **percent uncertainty** (%unc) is defined to be

$$\% \text{ unc} = \frac{\delta A}{A} \times 100\%. \tag{1.8}$$

Example 1.2 Calculating Percent Uncertainty: A Bag of Apples

A grocery store sells 5-lb bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

- Week 1 weight: 4.8 lb
- Week 2 weight: 5.3 lb
- Week 3 weight: 4.9 lb
- Week 4 weight: 5.4 lb

You determine that the weight of the 5-lb bag has an uncertainty of $\pm 0.4\text{ lb}$. What is the percent uncertainty of the bag's weight?

Strategy

First, observe that the expected value of the bag's weight, A, is 5 lb. The uncertainty in this value, δA, is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight:

$$\%\text{ unc} = \frac{\delta A}{A} \times 100\%. \tag{1.9}$$

Solution

Plug the known values into the equation:

$$\%\text{ unc} = \frac{0.4\text{ lb}}{5\text{ lb}} \times 100\% = 8\%. \tag{1.10}$$

Discussion

We can conclude that the weight of the apple bag is $5\text{ lb} \pm 8\%$. Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100%. If you do not do this, you will have a decimal quantity, not a percent value.

Uncertainties in Calculations

There is an uncertainty in anything calculated from measured quantities. For example, the area of a floor calculated from measurements of its length and width has an uncertainty because the length and width have uncertainties. How big is the uncertainty in something you calculate by multiplication or division? If the measurements going into the calculation have small uncertainties (a few percent or less), then the **method of adding percents** can be used for multiplication or division. This method says that *the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation*. For example, if a floor has a length of 4.00 m and a width of 3.00 m, with uncertainties of 2% and 1%, respectively, then the area of the floor is 12.0 m^2 and has an uncertainty of 3%. (Expressed as an area this is 0.36 m^2, which we round to 0.4 m^2 since the area of the floor is given to a tenth of a square meter.)

Check Your Understanding

A high school track coach has just purchased a new stopwatch. The stopwatch manual states that the stopwatch has an uncertainty of $\pm 0.05\text{ s}$. Runners on the track coach's team regularly clock 100-m sprints of 11.49 s to 15.01 s. At the school's last track meet, the first-place sprinter came in at 12.04 s and the second-place sprinter came in at 12.07 s. Will the coach's new stopwatch be helpful in timing the sprint team? Why or why not?

Solution

No, the uncertainty in the stopwatch is too great to effectively differentiate between the sprint times.

Precision of Measuring Tools and Significant Figures

An important factor in the accuracy and precision of measurements involves the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, a standard ruler can measure length to the nearest millimeter, while a caliper can measure length to the nearest 0.01 millimeter. The caliper is a more precise measuring tool because it can measure extremely small differences in length. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool. For example, if you use a standard ruler to measure the length of a stick, you may measure it to be 36.7 cm. You could not express this value as 36.71 cm because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between 36.6 cm and 36.7 cm, and he or she must estimate the value of the last digit. Using the method of **significant figures**, the rule is that *the last digit written down in a measurement is the first digit with some uncertainty*. In order to determine the number of significant digits in a value, start with the first measured value at the left and count the number of digits through the last digit written on the right. For example, the measured value 36.7 cm has three digits, or significant figures. Significant figures indicate the precision of a measuring tool that was used to measure a value.

Zeros

Special consideration is given to zeros when counting significant figures. The zeros in 0.053 are not significant, because they are only placekeepers that locate the decimal point. There are two significant figures in 0.053. The zeros in 10.053 are not placekeepers but are significant—this number has five significant figures. The zeros in 1300 may or may not be significant depending on the style of writing numbers. They could mean the number is known to the last digit, or they could be placekeepers. So 1300 could have two, three, or four significant figures. (To avoid this ambiguity, write 1300 in scientific notation.) *Zeros are significant except when they serve only as placekeepers*.

Check Your Understanding

Determine the number of significant figures in the following measurements:

a. 0.0009
b. 15,450.0
c. 6×10^3
d. 87.990
e. 30.42

Solution

(a) 1; the zeros in this number are placekeepers that indicate the decimal point

(b) 6; here, the zeros indicate that a measurement was made to the 0.1 decimal point, so the zeros are significant

(c) 1; the value 10^3 signifies the decimal place, not the number of measured values

(d) 5; the final zero indicates that a measurement was made to the 0.001 decimal point, so it is significant

(e) 4; any zeros located in between significant figures in a number are also significant

Significant Figures in Calculations

When combining measurements with different degrees of accuracy and precision, *the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value*. There are two different rules, one for multiplication and division and the other for addition and subtraction, as discussed below.

1. For multiplication and division: *The result should have the same number of significant figures as the quantity having the least significant figures entering into the calculation*. For example, the area of a circle can be calculated from its radius using $A = \pi r^2$. Let us see how many significant figures the area has if the radius has only two—say, $r = 1.2\text{ m}$. Then,

$$A = \pi r^2 = (3.1415927...)\times(1.2\text{ m})^2 = 4.5238934\text{ m}^2 \tag{1.11}$$

is what you would get using a calculator that has an eight-digit output. But because the radius has only two significant figures, it limits the calculated quantity to two significant figures or

$$A=4.5\text{ m}^2, \tag{1.12}$$

even though π is good to at least eight digits.

2. For addition and subtraction: *The answer can contain no more decimal places than the least precise measurement.* Suppose that you buy 7.56-kg of potatoes in a grocery store as measured with a scale with precision 0.01 kg. Then you drop off 6.052-kg of potatoes at your laboratory as measured by a scale with precision 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with precision 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:

$$\begin{array}{r} 7.56\ \ \text{kg} \\ -\ 6.052\text{ kg} \\ \underline{+13.7\ \ \ \text{kg}} \\ 15.208\text{ kg} \end{array} = 15.2\text{ kg}. \tag{1.13}$$

Next, we identify the least precise measurement: 13.7 kg. This measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer is rounded to the tenths place, giving us 15.2 kg.

Significant Figures in this Text

In this text, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. You will note that an answer given to three digits is based on input good to at least three digits, for example. If the input has fewer significant figures, the answer will also have fewer significant figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, particularly in optics, more accurate numbers are needed and more than three significant figures will be used. Finally, if a number is *exact*, such as the two in the formula for the circumference of a circle, $c = 2\pi r$, it does not affect the number of significant figures in a calculation.

Check Your Understanding

Perform the following calculations and express your answer using the correct number of significant digits.

(a) A woman has two bags weighing 13.5 pounds and one bag with a weight of 10.2 pounds. What is the total weight of the bags?

(b) The force F on an object is equal to its mass m multiplied by its acceleration a. If a wagon with mass 55 kg accelerates at a rate of $0.0255\ \text{m/s}^2$, what is the force on the wagon? (The unit of force is called the newton, and it is expressed with the symbol N.)

Solution

(a) 37.2 pounds; Because the number of bags is an exact value, it is not considered in the significant figures.

(b) 1.4 N; Because the value 55 kg has only two significant figures, the final value must also contain two significant figures.

PhET Explorations: Estimation

Explore size estimation in one, two, and three dimensions! Multiple levels of difficulty allow for progressive skill improvement.

PhET Interactive Simulation

Figure 1.26 Estimation (http://cnx.org/content/m42120/1.7/estimation_en.jar)

1.4 Approximation

On many occasions, physicists, other scientists, and engineers need to make **approximations** or "guesstimates" for a particular quantity. What is the distance to a certain destination? What is the approximate density of a given item? About how large a current will there be in a circuit? Many approximate numbers are based on formulae in which the input quantities are known only to a limited accuracy. As you develop problem-solving skills (that can be applied to a variety of fields through a study of physics), you will also develop skills at approximating. You will develop these skills through thinking more quantitatively, and by being willing to take risks. As with any endeavor, experience helps, as well as familiarity with units. These approximations allow us to rule out certain scenarios or unrealistic numbers. Approximations also allow us to challenge others and guide us in our approaches to our scientific world. Let us do two examples to illustrate this concept.

Example 1.3 Approximate the Height of a Building

Can you approximate the height of one of the buildings on your campus, or in your neighborhood? Let us make an approximation based upon the height of a person. In this example, we will calculate the height of a 39-story building.

Strategy

Think about the average height of an adult male. We can approximate the height of the building by scaling up from the height of a person.

Solution

Based on information in the example, we know there are 39 stories in the building. If we use the fact that the height of one story is approximately equal to about the length of two adult humans (each human is about 2-m tall), then we can estimate the total height of the building to be

$$\frac{2\text{ m}}{1\text{ person}}\times\frac{2\text{ person}}{1\text{ story}}\times 39\text{ stories} = 156\text{ m.} \tag{1.14}$$

Discussion

You can use known quantities to determine an approximate measurement of unknown quantities. If your hand measures 10 cm across, how many hand lengths equal the width of your desk? What other measurements can you approximate besides length?

Example 1.4 Approximating Vast Numbers: a Trillion Dollars

Figure 1.27 A bank stack contains one-hundred $100 bills, and is worth $10,000. How many bank stacks make up a trillion dollars? (credit: Andrew Magill)

The U.S. federal deficit in the 2008 fiscal year was a little greater than $10 trillion. Most of us do not have any concept of how much even one trillion actually is. Suppose that you were given a trillion dollars in $100 bills. If you made 100-bill stacks and used them to evenly cover a football field (between the end zones), make an approximation of how high the money pile would become. (We will use feet/inches rather than meters here because football fields are measured in yards.) One of your friends says 3 in., while another says 10 ft. What do you think?

Strategy

When you imagine the situation, you probably envision thousands of small stacks of 100 wrapped $100 bills, such as you might see in movies or at a bank. Since this is an easy-to-approximate quantity, let us start there. We can find the volume of a stack of 100 bills, find out how many stacks make up one trillion dollars, and then set this volume equal to the area of the football field multiplied by the unknown height.

Solution

(1) Calculate the volume of a stack of 100 bills. The dimensions of a single bill are approximately 3 in. by 6 in. A stack of 100 of these is about 0.5 in. thick. So the total volume of a stack of 100 bills is:

$$\begin{aligned}\text{volume of stack} &= \text{length}\times\text{width}\times\text{height},\\ \text{volume of stack} &= 6\text{ in.}\times 3\text{ in.}\times 0.5\text{ in.},\\ \text{volume of stack} &= 9\text{ in.}^3.\end{aligned} \tag{1.15}$$

(2) Calculate the number of stacks. Note that a trillion dollars is equal to $\$1\times10^{12},$ and a stack of one-hundred $\$100$ bills is equal to $\$10{,}000,$ or $\$1\times10^4$. The number of stacks you will have is:

$$\$1\times10^{12}(\text{a trillion dollars})/\ \$1\times10^4\text{ per stack} = 1\times10^8\text{ stacks.} \tag{1.16}$$

(3) Calculate the area of a football field in square inches. The area of a football field is $100\text{ yd}\times50\text{ yd},$ which gives $5{,}000\text{ yd}^2.$ Because we are working in inches, we need to convert square yards to square inches:

$$\begin{aligned}\text{Area} &= 5{,}000\text{ yd}^2\times\frac{3\text{ ft}}{1\text{ yd}}\times\frac{3\text{ ft}}{1\text{ yd}}\times\frac{12\text{ in.}}{1\text{ ft}}\times\frac{12\text{ in.}}{1\text{ ft}} = 6{,}480{,}000\text{ in.}^2,\\ \text{Area} &\approx 6\times10^6\text{ in.}^2.\end{aligned} \tag{1.17}$$

This conversion gives us $6\times10^6\text{ in.}^2$ for the area of the field. (Note that we are using only one significant figure in these calculations.)

(4) Calculate the total volume of the bills. The volume of all the $\$100$ -bill stacks is $9\text{ in.}^3/\text{stack}\times10^8\text{ stacks} = 9\times10^8\text{ in.}^3.$

(5) Calculate the height. To determine the height of the bills, use the equation:

$$\begin{aligned}\text{volume of bills} &= \text{ar}\qquad\qquad\times\text{height of money:}\\ \text{Height of money} &= \frac{\text{volume of bills}}{\text{ar}},\\ \text{Height of money} &= \frac{9\times10^8\text{in.}^3}{6\times10^6\text{in.}^2} = 1.33\times10^2\text{in.},\\ \text{Height of money} &\approx 1\times10^2\text{in.} = 100\text{ in.}\end{aligned} \tag{1.18}$$

The height of the money will be about 100 in. high. Converting this value to feet gives

$$100\text{ in.}\times\frac{1\text{ ft}}{12\text{ in.}} = 8.33\text{ ft}\approx 8\text{ ft.} \tag{1.19}$$

Discussion

The final approximate value is much higher than the early estimate of 3 in., but the other early estimate of 10 ft (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise tell you in terms of rough “guesstimates” versus carefully calculated approximations?

Check Your Understanding

Using mental math and your understanding of fundamental units, approximate the area of a regulation basketball court. Describe the process you used to arrive at your final approximation.

Solution

An average male is about two meters tall. It would take approximately 15 men laid out end to end to cover the length, and about 7 to cover the width. That gives an approximate area of $420\ \text{m}^2$.

Glossary

accuracy: the degree to which a measured value agrees with correct value for that measurement

approximation: an estimated value based on prior experience and reasoning

classical physics: physics that was developed from the Renaissance to the end of the 19th century

conversion factor: a ratio expressing how many of one unit are equal to another unit

derived units: units that can be calculated using algebraic combinations of the fundamental units

English units: system of measurement used in the United States; includes units of measurement such as feet, gallons, and pounds

fundamental units: units that can only be expressed relative to the procedure used to measure them

kilogram: the SI unit for mass, abbreviated (kg)

law: a description, using concise language or a mathematical formula, a generalized pattern in nature that is supported by scientific evidence and repeated experiments

meter: the SI unit for length, abbreviated (m)

method of adding percents: the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation

metric system: a system in which values can be calculated in factors of 10

model: representation of something that is often too difficult (or impossible) to display directly

modern physics: the study of relativity, quantum mechanics, or both

order of magnitude: refers to the size of a quantity as it relates to a power of 10

percent uncertainty: the ratio of the uncertainty of a measurement to the measured value, expressed as a percentage

physical quantity : a characteristic or property of an object that can be measured or calculated from other measurements

physics: the science concerned with describing the interactions of energy, matter, space, and time; it is especially interested in what fundamental mechanisms underlie every phenomenon

precision: the degree to which repeated measurements agree with each other

quantum mechanics: the study of objects smaller than can be seen with a microscope

relativity: the study of objects moving at speeds greater than about 1% of the speed of light, or of objects being affected by a strong gravitational field

SI units : the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams

scientific method: a method that typically begins with an observation and question that the scientist will research; next, the scientist typically performs some research about the topic and then devises a hypothesis; then, the scientist will test the hypothesis by performing an experiment; finally, the scientist analyzes the results of the experiment and draws a conclusion

second: the SI unit for time, abbreviated (s)

significant figures: express the precision of a measuring tool used to measure a value

theory: an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

uncertainty: a quantitative measure of how much your measured values deviate from a standard or expected value

units : a standard used for expressing and comparing measurements

Section Summary

1.1 Physics: An Introduction

- Science seeks to discover and describe the underlying order and simplicity in nature.
- Physics is the most basic of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.

1.2 Physical Quantities and Units

- Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.
- Units are standards for expressing and comparing the measurement of physical quantities. All units can be expressed as combinations of four fundamental units.
- The four fundamental units we will use in this text are the meter (for length), the kilogram (for mass), the second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.
- The four fundamental units are abbreviated as follows: meter, m; kilogram, kg; second, s; and ampere, A. The metric system also uses a standard set of prefixes to denote each order of magnitude greater than or lesser than the fundamental unit itself.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.

1.3 Accuracy, Precision, and Significant Figures

- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.
- The precision of a *measuring tool* is related to the size of its measurement increments. The smaller the measurement increment, the more precise the tool.
- Significant figures express the precision of a measuring tool.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.

1.4 Approximation

Scientists often approximate the values of quantities to perform calculations and analyze systems.

Conceptual Questions

1.1 Physics: An Introduction

1. Models are particularly useful in relativity and quantum mechanics, where conditions are outside those normally encountered by humans. What is a model?

2. How does a model differ from a theory?

3. If two different theories describe experimental observations equally well, can one be said to be more valid than the other (assuming both use accepted rules of logic)?

4. What determines the validity of a theory?

5. Certain criteria must be satisfied if a measurement or observation is to be believed. Will the criteria necessarily be as strict for an expected result as for an unexpected result?

6. Can the validity of a model be limited, or must it be universally valid? How does this compare to the required validity of a theory or a law?

7. Classical physics is a good approximation to modern physics under certain circumstances. What are they?

8. When is it *necessary* to use relativistic quantum mechanics?

9. Can classical physics be used to accurately describe a satellite moving at a speed of 7500 m/s? Explain why or why not.

1.2 Physical Quantities and Units

10. Identify some advantages of metric units.

1.3 Accuracy, Precision, and Significant Figures

11. What is the relationship between the accuracy and uncertainty of a measurement?

12. Prescriptions for vision correction are given in units called *diopters* (D). Determine the meaning of that unit. Obtain information (perhaps by calling an optometrist or performing an internet search) on the minimum uncertainty with which corrections in diopters are determined and the accuracy with which corrective lenses can be produced. Discuss the sources of uncertainties in both the prescription and accuracy in the manufacture of lenses.

Problems & Exercises

1.2 Physical Quantities and Units

1. The speed limit on some interstate highways is roughly 100 km/h. (a) What is this in meters per second? (b) How many miles per hour is this?

2. A car is traveling at a speed of 33 m/s. (a) What is its speed in kilometers per hour? (b) Is it exceeding the 90 km/h speed limit?

3. Show that $1.0\text{ m/s} = 3.6\text{ km/h}$. Hint: Show the explicit steps involved in converting $1.0\text{ m/s} = 3.6\text{ km/h}.$

4. American football is played on a 100-yd-long field, excluding the end zones. How long is the field in meters? (Assume that 1 meter equals 3.281 feet.)

5. Soccer fields vary in size. A large soccer field is 115 m long and 85 m wide. What are its dimensions in feet and inches? (Assume that 1 meter equals 3.281 feet.)

6. What is the height in meters of a person who is 6 ft 1.0 in. tall? (Assume that 1 meter equals 39.37 in.)

7. Mount Everest, at 29,028 feet, is the tallest mountain on the Earth. What is its height in kilometers? (Assume that 1 kilometer equals 3,281 feet.)

8. The speed of sound is measured to be 342 m/s on a certain day. What is this in km/h?

9. Tectonic plates are large segments of the Earth's crust that move slowly. Suppose that one such plate has an average speed of 4.0 cm/year. (a) What distance does it move in 1 s at this speed? (b) What is its speed in kilometers per million years?

10. (a) Refer to Table 1.3 to determine the average distance between the Earth and the Sun. Then calculate the average speed of the Earth in its orbit in kilometers per second. (b) What is this in meters per second?

1.3 Accuracy, Precision, and Significant Figures

Express your answers to problems in this section to the correct number of significant figures and proper units.

11. Suppose that your bathroom scale reads your mass as 65 kg with a 3% uncertainty. What is the uncertainty in your mass (in kilograms)?

12. A good-quality measuring tape can be off by 0.50 cm over a distance of 20 m. What is its percent uncertainty?

13. (a) A car speedometer has a 5.0% uncertainty. What is the range of possible speeds when it reads 90 km/h? (b) Convert this range to miles per hour. $(1\text{ km} = 0.6214\text{ mi})$

14. An infant's pulse rate is measured to be 130 ± 5 beats/min. What is the percent uncertainty in this measurement?

15. (a) Suppose that a person has an average heart rate of 72.0 beats/min. How many beats does he or she have in 2.0 y? (b) In 2.00 y? (c) In 2.000 y?

16. A can contains 375 mL of soda. How much is left after 308 mL is removed?

17. State how many significant figures are proper in the results of the following calculations: (a) $(106.7)(98.2) / (46.210)(1.01)$ (b) $(18.7)^2$ (c) $\left(1.60\times10^{-19}\right)(3712)$.

18. (a) How many significant figures are in the numbers 99 and 100? (b) If the uncertainty in each number is 1, what is the percent uncertainty in each? (c) Which is a more meaningful way to express the accuracy of these two numbers, significant figures or percent uncertainties?

19. (a) If your speedometer has an uncertainty of 2.0 km/h at a speed of 90 km/h, what is the percent uncertainty? (b) If it has the same percent uncertainty when it reads 60 km/h, what is the range of speeds you could be going?

20. (a) A person's blood pressure is measured to be $120 \pm 2\text{ mm Hg}$. What is its percent uncertainty? (b) Assuming the same percent uncertainty, what is the uncertainty in a blood pressure measurement of 80 mm Hg?

21. A person measures his or her heart rate by counting the number of beats in 30 s. If 40 ± 1 beats are counted in $30.0 \pm 0.5\text{ s}$, what is the heart rate and its uncertainty in beats per minute?

22. What is the area of a circle 3.102 cm in diameter?

23. If a marathon runner averages 9.5 mi/h, how long does it take him or her to run a 26.22-mi marathon?

24. A marathon runner completes a 42.188-km course in 2 h, 30 min, and 12 s. There is an uncertainty of 25 m in the distance traveled and an uncertainty of 1 s in the elapsed time. (a) Calculate the percent uncertainty in the distance. (b) Calculate the uncertainty in the elapsed time. (c) What is the average speed in meters per second? (d) What is the uncertainty in the average speed?

25. The sides of a small rectangular box are measured to be $1.80 \pm 0.01\text{ cm}$, $2.05 \pm 0.02\text{ cm, and } 3.1 \pm 0.1\text{ cm}$ long. Calculate its volume and uncertainty in cubic centimeters.

26. When non-metric units were used in the United Kingdom, a unit of mass called the *pound-mass* (lbm) was employed, where $1\text{ lbm} = 0.4539\text{ kg}$. (a) If there is an uncertainty of 0.0001 kg in the pound-mass unit, what is its percent uncertainty? (b) Based on that percent uncertainty, what mass in pound-mass has an uncertainty of 1 kg when converted to kilograms?

27. The length and width of a rectangular room are measured to be $3.955 \pm 0.005\text{ m}$ and $3.050 \pm 0.005\text{ m}$. Calculate the area of the room and its uncertainty in square meters.

28. A car engine moves a piston with a circular cross section of $7.500 \pm 0.002\text{ cm}$ diameter a distance of $3.250 \pm 0.001\text{ cm}$ to compress the gas in the cylinder. (a) By what amount is the gas decreased in volume in cubic centimeters? (b) Find the uncertainty in this volume.

1.4 Approximation

29. How many heartbeats are there in a lifetime?

30. A generation is about one-third of a lifetime. Approximately how many generations have passed since the year 0 AD?

31. How many times longer than the mean life of an extremely unstable atomic nucleus is the lifetime of a human? (Hint: The lifetime of an unstable atomic nucleus is on the order of 10^{-22} s.)

32. Calculate the approximate number of atoms in a bacterium. Assume that the average mass of an atom in the bacterium is ten times the mass of a hydrogen atom. (Hint: The mass of a hydrogen atom is on the order of 10^{-27} kg and the mass of a bacterium is on the order of 10^{-15} kg.)

Figure 1.28 This color-enhanced photo shows *Salmonella typhimurium* (red) attacking human cells. These bacteria are commonly known for causing foodborne illness. Can you estimate the number of atoms in each bacterium? (credit: Rocky Mountain Laboratories, NIAID, NIH)

33. Approximately how many atoms thick is a cell membrane, assuming all atoms there average about twice the size of a hydrogen atom?

34. (a) What fraction of Earth's diameter is the greatest ocean depth? (b) The greatest mountain height?

35. (a) Calculate the number of cells in a hummingbird assuming the mass of an average cell is ten times the mass of a bacterium. (b) Making the same assumption, how many cells are there in a human?

36. Assuming one nerve impulse must end before another can begin, what is the maximum firing rate of a nerve in impulses per second?

2 KINEMATICS

Figure 2.1 The motion of an American kestrel through the air can be described by the bird's displacement, speed, velocity, and acceleration. When it flies in a straight line without any change in direction, its motion is said to be one dimensional. (credit: Vince Maidens, Wikimedia Commons)

Learning Objectives

2.1. Displacement
- Define position, displacement, distance, and distance traveled.
- Explain the relationship between position and displacement.
- Distinguish between displacement and distance traveled.
- Calculate displacement and distance given initial position, final position, and the path between the two.

2.2. Vectors, Scalars, and Coordinate Systems
- Define and distinguish between scalar and vector quantities.
- Assign a coordinate system for a scenario involving one-dimensional motion.

2.3. Time, Velocity, and Speed
- Explain the relationships between instantaneous velocity, average velocity, instantaneous speed, average speed, displacement, and time.
- Calculate velocity and speed given initial position, initial time, final position, and final time.
- Derive a graph of velocity vs. time given a graph of position vs. time.
- Interpret a graph of velocity vs. time.

2.4. Acceleration
- Define and distinguish between instantaneous acceleration, average acceleration, and deceleration.
- Calculate acceleration given initial time, initial velocity, final time, and final velocity.

2.5. Motion Equations for Constant Acceleration in One Dimension
- Calculate displacement of an object that is not accelerating, given initial position and velocity.
- Calculate final velocity of an accelerating object, given initial velocity, acceleration, and time.
- Calculate displacement and final position of an accelerating object, given initial position, initial velocity, time, and acceleration.

2.6. Problem-Solving Basics for One-Dimensional Kinematics
- Apply problem-solving steps and strategies to solve problems of one-dimensional kinematics.
- Apply strategies to determine whether or not the result of a problem is reasonable, and if not, determine the cause.

2.7. Falling Objects
- Describe the effects of gravity on objects in motion.
- Describe the motion of objects that are in free fall.
- Calculate the position and velocity of objects in free fall.

2.8. Graphical Analysis of One-Dimensional Motion
- Describe a straight-line graph in terms of its slope and *y*-intercept.
- Determine average velocity or instantaneous velocity from a graph of position vs. time.
- Determine average or instantaneous acceleration from a graph of velocity vs. time.
- Derive a graph of velocity vs. time from a graph of position vs. time.
- Derive a graph of acceleration vs. time from a graph of velocity vs. time.

Introduction to One-Dimensional Kinematics

Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. And even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: *How long will it take for a space probe to get to Mars? Where will a football land if it is thrown at a certain angle?* But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

Our formal study of physics begins with **kinematics** which is defined as the *study of motion without considering its causes*. The word “kinematics” comes from a Greek term meaning motion and is related to other English words such as “cinema” (movies) and “kinesiology” (the study of human motion). In one-dimensional kinematics and **Two-Dimensional Kinematics** we will study only the *motion* of a football, for example, without worrying about what forces cause or change its motion. Such considerations come in other chapters. In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion. In **Two-Dimensional Kinematics**, we apply concepts developed here to study motion along curved paths (two- and three-dimensional motion); for example, that of a car rounding a curve.

2.1 Displacement

Figure 2.2 These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference. (credit: Suzan Black, Fotopedia)

Position

In order to describe the motion of an object, you must first be able to describe its **position**—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor’s position could be described in terms of where she is in relation to the nearby white board. (See **Figure 2.3**.) In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame. (See **Figure 2.4**.)

Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object’s position changes. This change in position is known as **displacement**. The word “displacement” implies that an object has moved, or has been displaced.

Displacement

Displacement is the *change in position* of an object:

$$\Delta x = x_\text{f} - x_0, \tag{2.1}$$

where Δx is displacement, x_f is the final position, and x_0 is the initial position.

In this text the upper case Greek letter Δ (delta) always means “change in” whatever quantity follows it; thus, Δx means *change in position*. Always solve for displacement by subtracting initial position x_0 from final position x_f .

Note that the SI unit for displacement is the meter (m) (see **Physical Quantities and Units**), but sometimes kilometers, miles, feet, and other units of length are used. Keep in mind that when units other than the meter are used in a problem, you may need to convert them into meters to complete the calculation.

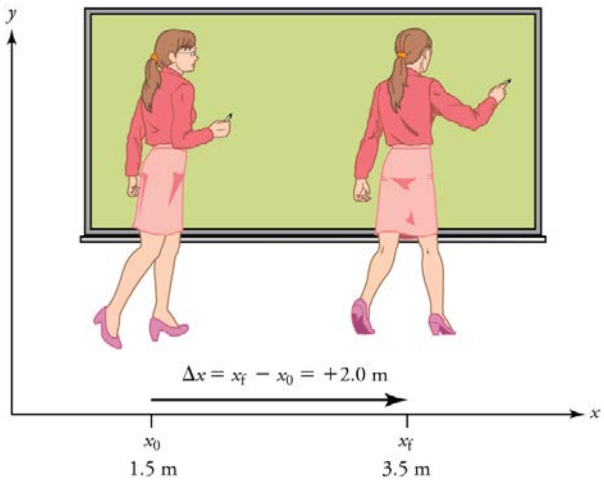


Figure 2.3 A professor paces left and right while lecturing. Her position relative to Earth is given by x. The $+2.0\text{ m}$ displacement of the professor relative to Earth is represented by an arrow pointing to the right.

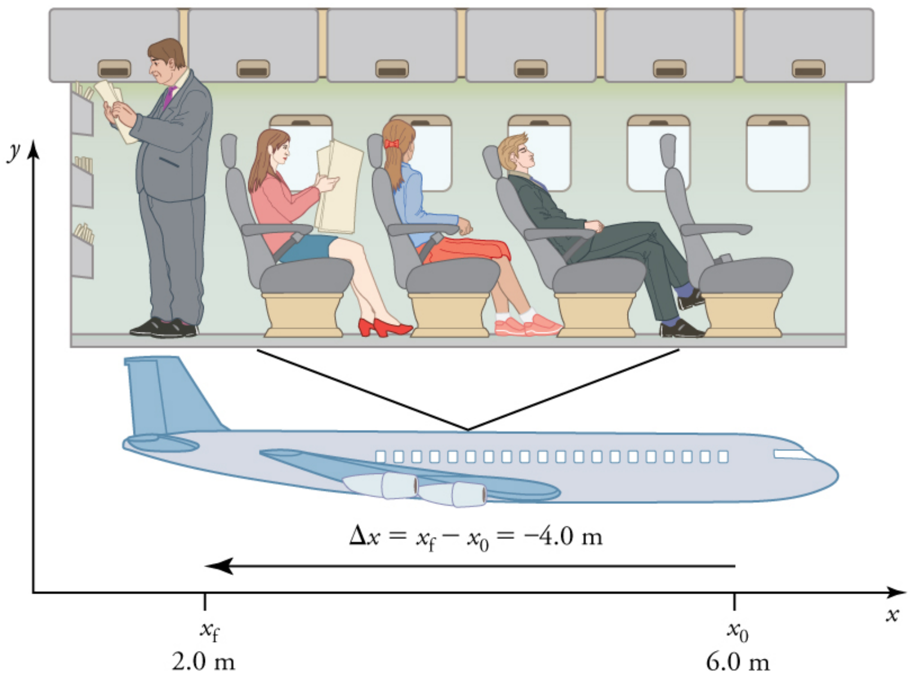


Figure 2.4 A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The -4.0-m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far) in **Figure 2.3**.

Note that displacement has a direction as well as a magnitude. The professor's displacement is 2.0 m to the right, and the airline passenger's displacement is 4.0 m toward the rear. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor's initial position is $x_0 = 1.5\text{ m}$ and her final position is $x_\text{f} = 3.5\text{ m}$. Thus her displacement is

$$\Delta x = x_\text{f} - x_0 = 3.5\text{ m} - 1.5\text{ m} = +2.0\text{ m}. \tag{2.2}$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is $x_0 = 6.0\text{ m}$ and his final position is $x_\text{f} = 2.0\text{ m}$, so his displacement is

$$\Delta x = x_\text{f} - x_0 = 2.0\text{ m} - 6.0\text{ m} = -4.0\text{ m}. \tag{2.3}$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative x direction in our coordinate system.

Distance

Although displacement is described in terms of direction, distance is not. **Distance** is defined to be *the magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. **Distance traveled** is *the total length of the path traveled between two positions*. Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

Misconception Alert: Distance Traveled vs. Magnitude of Displacement

It is important to note that the *distance traveled*, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

Check Your Understanding

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?

Solution

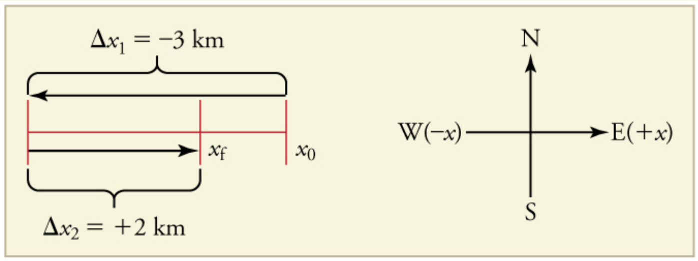


Figure 2.5

(a) The rider's displacement is $\Delta x = x_\mathrm{f} - x_0 = -1\text{ km}$. (The displacement is negative because we take east to be positive and west to be negative.)

(b) The distance traveled is $3\text{ km} + 2\text{ km} = 5\text{ km}$.

(c) The magnitude of the displacement is 1 km.

2.2 Vectors, Scalars, and Coordinate Systems

Figure 2.6 The motion of this Eclipse Concept jet can be described in terms of the distance it has traveled (a scalar quantity) or its displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. In this case, it may be convenient to choose motion toward the left as positive motion (it is the forward direction for the plane), although in many cases, the x-coordinate runs from left to right, with motion to the right as positive and motion to the left as negative. (credit: Armchair Aviator, Flickr)

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A **vector** is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

The direction of a vector in one-dimensional motion is given simply by a plus $(+)$ or minus $(-)$ sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A **scalar** is any quantity that has a magnitude, but no direction. For example, a 20°C temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a -20°C temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in **Figure 2.6**, it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.

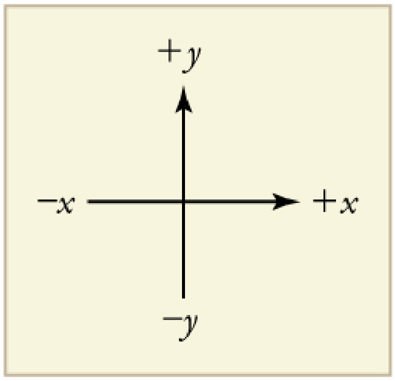


Figure 2.7 It is usually convenient to consider motion upward or to the right as positive $(+)$ and motion downward or to the left as negative $(-)$.

Check Your Understanding

A person's speed can stay the same as he or she rounds a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

Solution
Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

2.3 Time, Velocity, and Speed

Figure 2.8 The motion of these racing snails can be described by their speeds and their velocities. (credit: tobitasflickr, Flickr)

There is more to motion than distance and displacement. Questions such as, "How long does a foot race take?" and "What was the runner's speed?" cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

Time

As discussed in **Physical Quantities and Units**, the most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of time is simple— **time** is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s. We might, for example, observe that a certain pendulum makes one full swing every 0.75 s. We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time** $\boldsymbol{\Delta t}$ is the difference between the ending time and beginning time,

$$\Delta t = t_\text{f} - t_0, \tag{2.4}$$

where Δt is the change in time or elapsed time, t_f is the time at the end of the motion, and t_0 is the time at the beginning of the motion. (As usual, the delta symbol, Δ, means the change in the quantity that follows it.)

Life is simpler if the beginning time t_0 is taken to be zero, as when we use a stopwatch. If we were using a stopwatch, it would simply read zero at the start of the lecture and 50 min at the end. If $t_0 = 0$, then $\Delta t = t_\text{f} \equiv t$.

In this text, for simplicity's sake,

- motion starts at time equal to zero $(t_0 = 0)$
- the symbol t is used for elapsed time unless otherwise specified $(\Delta t = t_\text{f} \equiv t)$

Velocity

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

Average Velocity

Average velocity is *displacement (change in position) divided by the time of travel*,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_\text{f} - x_0}{t_\text{f} - t_0}, \tag{2.5}$$

where $\bar{v}$ is the *average* (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_f and x_0 are the final and beginning positions at times t_f and t_0, respectively. If the starting time t_0 is taken to be zero, then the average velocity is simply

$$\bar{v} = \frac{\Delta x}{t}. \tag{2.6}$$

Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move −4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be

$$\bar{v} = \frac{\Delta x}{t} = \frac{-4\text{ m}}{5\text{ s}} = -0.8\text{ m/s}. \tag{2.7}$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.

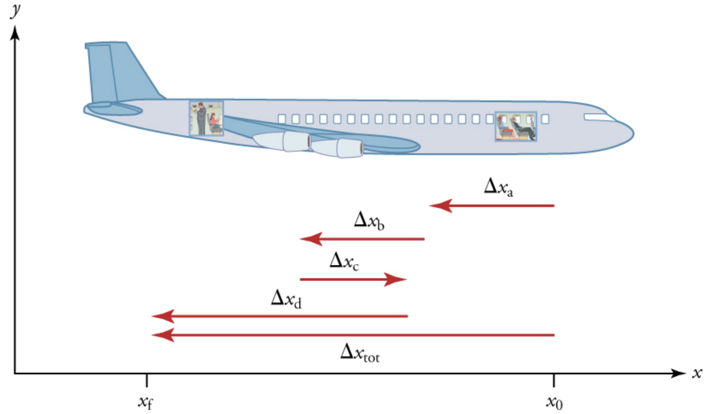


Figure 2.9 A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity* or the *velocity at a specific instant*. A car's speedometer, for example, shows the magnitude (but not the direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.) **Instantaneous velocity** v is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Mathematically, finding instantaneous velocity, v, at a precise instant t can involve taking a limit, a calculus operation beyond the scope of this text. However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

Speed

In everyday language, most people use the terms “speed” and “velocity” interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

Instantaneous speed is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of −3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. **Average speed** is the distance traveled divided by elapsed time.

We have noted that distance traveled can be greater than displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car’s odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero. (Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.

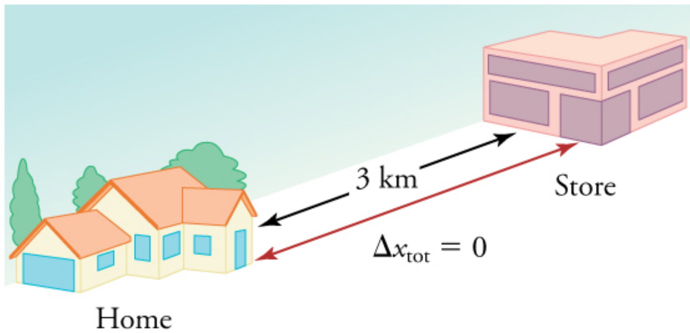


Figure 2.10 During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in **Figure 2.11**. (Note that these graphs depict a very simplified **model** of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we’ll probably stop at the store. But for simplicity’s sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)

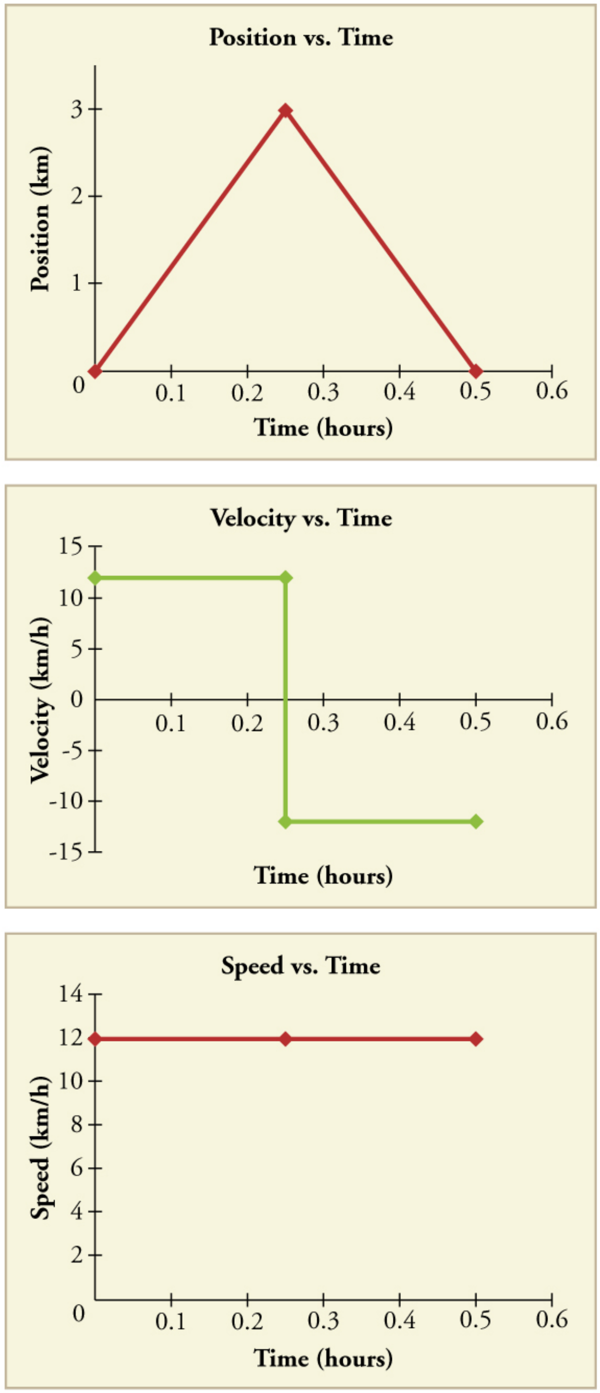


Figure 2.11 Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.

Making Connections: Take-Home Investigation—Getting a Sense of Speed

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
- determine the speed of an ant, snail, or falling leaf

Check Your Understanding

A commuter train travels from Baltimore to Washington, DC, and back in 1 hour and 45 minutes. The distance between the two stations is approximately 40 miles. What is (a) the average velocity of the train, and (b) the average speed of the train in m/s?

Solution

(a) The average velocity of the train is zero because $x_{\text{f}} = x_0$; the train ends up at the same place it starts.

(b) The average speed of the train is calculated below. Note that the train travels 40 miles one way and 40 miles back, for a total distance of 80 miles.

$$\frac{\text{distance}}{\text{time}} = \frac{80\text{ miles}}{105\text{ minutes}} \tag{2.8}$$

$$\frac{80\text{ miles}}{105\text{ minutes}} \times \frac{5280\text{ feet}}{1\text{ mile}} \times \frac{1\text{ meter}}{3.28\text{ feet}} \times \frac{1\text{ minute}}{60\text{ seconds}} = 20\text{ m/s} \tag{2.9}$$

2.4 Acceleration

Figure 2.12 A plane decelerates, or slows down, as it comes in for landing in St. Maarten. Its acceleration is opposite in direction to its velocity. (credit: Steve Conry, Flickr)

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the **acceleration**, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

Average Acceleration

Average Acceleration is *the rate at which velocity changes*,

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_{\text{f}} - v_0}{t_{\text{f}} - t_0}, \tag{2.10}$$

where $\bar{a}$ is average acceleration, v is velocity, and t is time. (The bar over the a means *average* acceleration.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are m/s^2 , meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in *direction*. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

Acceleration as a Vector

Acceleration is a vector in the same direction as the *change* in velocity, Δv . Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. When an object slows down, its acceleration is opposite to the direction of its motion. This is known as **deceleration**.

Figure 2.13 A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki, Flickr)

Misconception Alert: Deceleration vs. Negative Acceleration

Deceleration always refers to acceleration in the direction opposite to the direction of the velocity. Deceleration always reduces speed. Negative acceleration, however, is acceleration *in the negative direction in the chosen coordinate system*. Negative acceleration may or may not be deceleration, and deceleration may or may not be considered negative acceleration. For example, consider **Figure 2.14**.

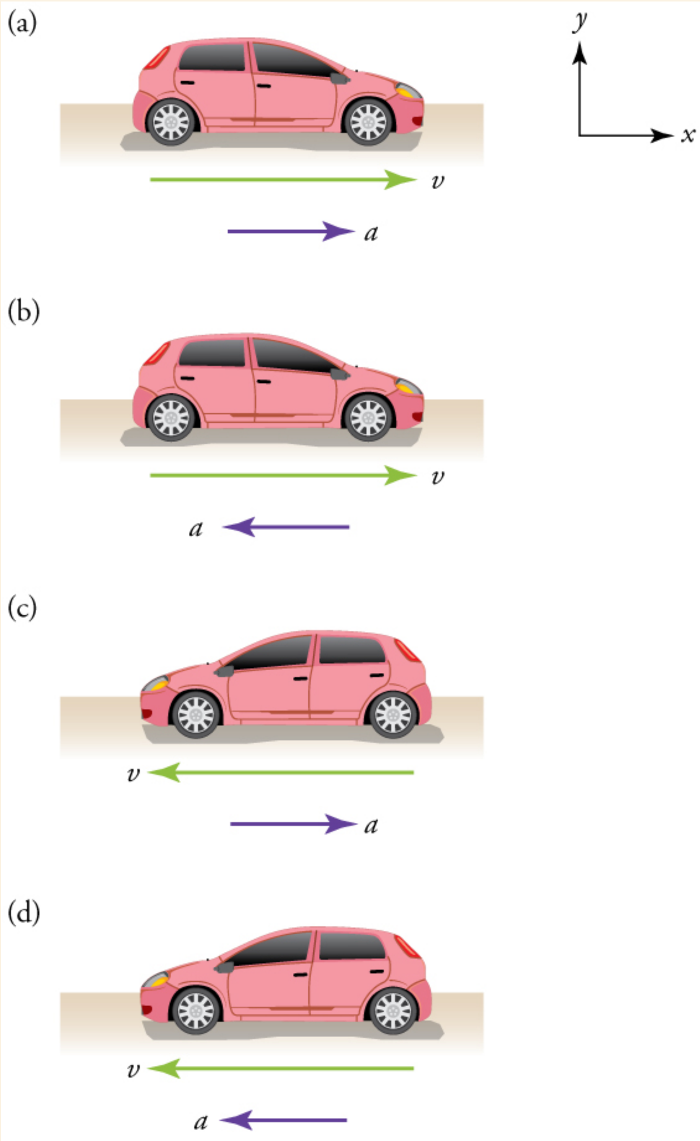


Figure 2.14 (a) This car is speeding up as it moves toward the right. It therefore has positive acceleration in our coordinate system. (b) This car is slowing down as it moves toward the right. Therefore, it has negative acceleration in our coordinate system, because its acceleration is toward the left. The car is also decelerating: the direction of its acceleration is opposite to its direction of motion. (c) This car is moving toward the left, but slowing down over time. Therefore, its acceleration is positive in our coordinate system because it is toward the right. However, the car is decelerating because its acceleration is opposite to its motion. (d) This car is speeding up as it moves toward the left. It has negative acceleration because it is accelerating toward the left. However, because its acceleration is in the same direction as its motion, it is speeding up (*not* decelerating).

Example 2.1 Calculating Acceleration: A Racehorse Leaves the Gate

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?

Figure 2.15 (credit: Jon Sullivan, PD Photo.org)

Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.

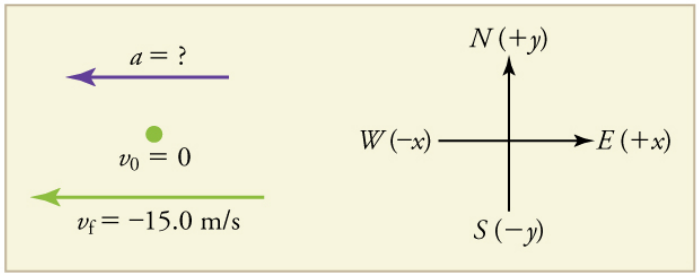


Figure 2.16

We can solve this problem by identifying Δv and Δt from the given information and then calculating the average acceleration directly from the equation $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_\text{f} - v_0}{t_\text{f} - t_0}$.

Solution

1. Identify the knowns. $v_0 = 0$, $v_\text{f} = -15.0\text{ m/s}$ (the negative sign indicates direction toward the west), $\Delta t = 1.80\text{ s}$.

2. Find the change in velocity. Since the horse is going from zero to -15.0 m/s, its change in velocity equals its final velocity: $\Delta v = v_\text{f} = -15.0\text{ m/s}$.

3. Plug in the known values (Δv and Δt) and solve for the unknown $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0\text{ m/s}}{1.80\text{ s}} = -8.33\text{ m/s}^2. \tag{2.11}$$

Discussion

The negative sign for acceleration indicates that acceleration is toward the west. An acceleration of 8.33 m/s^2 due west means that the horse increases its velocity by 8.33 m/s due west each second, that is, 8.33 meters per second per second, which we write as 8.33 m/s^2. This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

Instantaneous Acceleration

Instantaneous acceleration a, or the *acceleration at a specific instant in time*, is obtained by the same process as discussed for instantaneous velocity in **Time, Velocity, and Speed**—that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion. Figure 2.17 shows graphs of instantaneous acceleration versus time for two very different motions. In Figure 2.17(a), the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In this case, we should treat this motion as if it had a constant acceleration equal to the average (in this case about 1.8 m/s^2). In Figure 2.17(b), the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of $+3.0\text{ m/s}^2$ and -2.0 m/s^2, respectively.

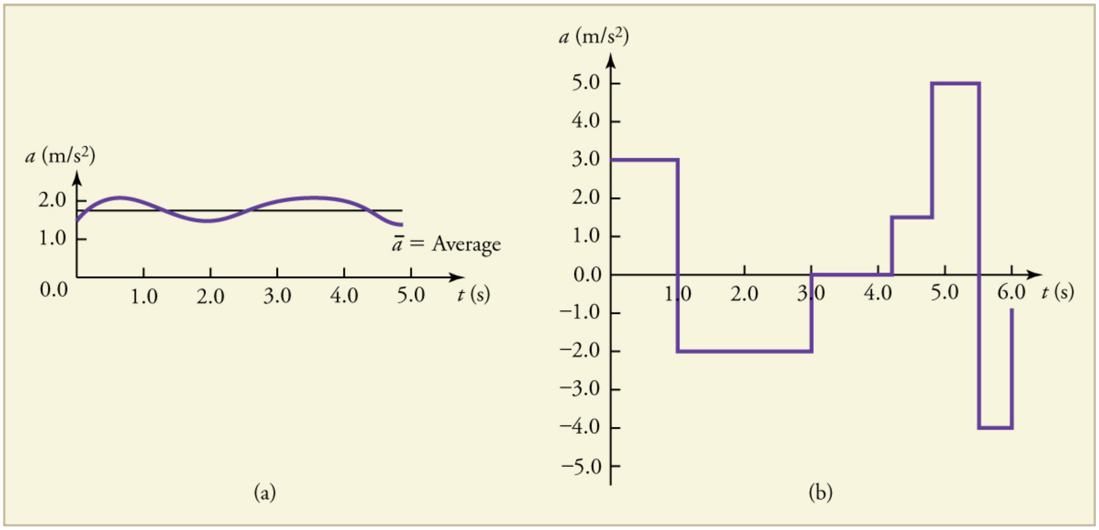


Figure 2.17 Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation.

The next several examples consider the motion of the subway train shown in **Figure 2.18**. In (a) the shuttle moves to the right, and in (b) it moves to the left. The examples are designed to further illustrate aspects of motion and to illustrate some of the reasoning that goes into solving problems.

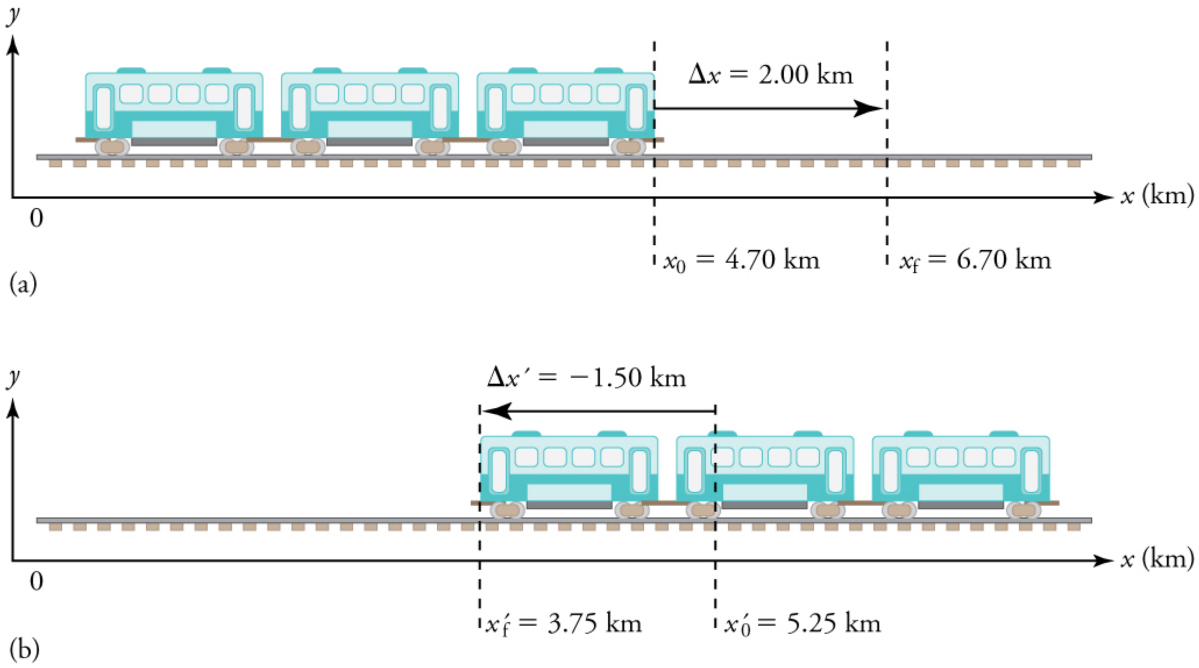


Figure 2.18 One-dimensional motion of a subway train considered in **Example 2.2**, **Example 2.3**, **Example 2.4**, **Example 2.5**, **Example 2.6**, and **Example 2.7**. Here we have chosen the x-axis so that + means to the right and $-$ means to the left for displacements, velocities, and accelerations. (a) The subway train moves to the right from x_0 to x_f. Its displacement Δx is +2.0 km. (b) The train moves to the left from x'_0 to x'_f. Its displacement $\Delta x'$ is -1.5 km. (Note that the prime symbol (′) is used simply to distinguish between displacement in the two different situations. The distances of travel and the size of the cars are on different scales to fit everything into the diagram.)

Example 2.2 Calculating Displacement: A Subway Train

What are the magnitude and sign of displacements for the motions of the subway train shown in parts (a) and (b) of **Figure 2.18**?

Strategy

A drawing with a coordinate system is already provided, so we don't need to make a sketch, but we should analyze it to make sure we understand what it is showing. Pay particular attention to the coordinate system. To find displacement, we use the equation $\Delta x = x_f - x_0$. This is straightforward since the initial and final positions are given.

Solution

1. Identify the knowns. In the figure we see that $x_f = 6.70 \text{ km}$ and $x_0 = 4.70 \text{ km}$ for part (a), and $x'_f = 3.75 \text{ km}$ and $x'_0 = 5.25 \text{ km}$ for part (b).

2. Solve for displacement in part (a).

$$\Delta x = x_f - x_0 = 6.70 \text{ km} - 4.70 \text{ km} = +2.00 \text{ km} \tag{2.12}$$

3. Solve for displacement in part (b).

$$\Delta x' = x'_f - x'_0 = 3.75 \text{ km} - 5.25 \text{ km} = -1.50 \text{ km} \tag{2.13}$$

Discussion

The direction of the motion in (a) is to the right and therefore its displacement has a positive sign, whereas motion in (b) is to the left and thus has a negative sign.

Example 2.3 Comparing Distance Traveled with Displacement: A Subway Train

What are the distances traveled for the motions shown in parts (a) and (b) of the subway train in **Figure 2.18**?

Strategy

To answer this question, think about the definitions of distance and distance traveled, and how they are related to displacement. Distance between two positions is defined to be the magnitude of displacement, which was found in **Example 2.2**. Distance traveled is the total length of the path traveled between the two positions. (See **Displacement**.) In the case of the subway train shown in **Figure 2.18**, the distance traveled is the same as the distance between the initial and final positions of the train.

Solution

1. The displacement for part (a) was +2.00 km. Therefore, the distance between the initial and final positions was 2.00 km, and the distance traveled was 2.00 km.

2. The displacement for part (b) was $-1.5 \text{ km}.$ Therefore, the distance between the initial and final positions was 1.50 km, and the distance traveled was 1.50 km.

Discussion

Distance is a scalar. It has magnitude but no sign to indicate direction.

Example 2.4 Calculating Acceleration: A Subway Train Speeding Up

Suppose the train in **Figure 2.18**(a) accelerates from rest to 30.0 km/h in the first 20.0 s of its motion. What is its average acceleration during that time interval?

Strategy

It is worth it at this point to make a simple sketch:

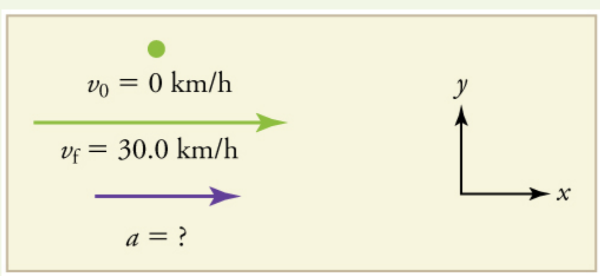


Figure 2.19

This problem involves three steps. First we must determine the change in velocity, then we must determine the change in time, and finally we use these values to calculate the acceleration.

Solution

1. Identify the knowns. $v_0 = 0$ (the trains starts at rest), $v_f = 30.0 \text{ km/h}$, and $\Delta t = 20.0 \text{ s}$.

2. Calculate Δv . Since the train starts from rest, its change in velocity is $\Delta v = +30.0 \text{ km/h}$, where the plus sign means velocity to the right.

3. Plug in known values and solve for the unknown, $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+30.0 \text{ km/h}}{20.0 \text{ s}} \tag{2.14}$$

4. Since the units are mixed (we have both hours and seconds for time), we need to convert everything into SI units of meters and seconds. (See **Physical Quantities and Units** for more guidance.)

$$\bar{a} = \left(\frac{+30\text{ km/h}}{20.0\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 0.417\text{ m/s}^2 \tag{2.15}$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train starts from rest and ends up with a velocity to the right (also positive). So acceleration is in the same direction as the *change* in velocity, as is always the case.

Example 2.5 Calculate Acceleration: A Subway Train Slowing Down

Now suppose that at the end of its trip, the train in **Figure 2.18**(a) slows to a stop from a speed of 30.0 km/h in 8.00 s. What is its average acceleration while stopping?

Strategy

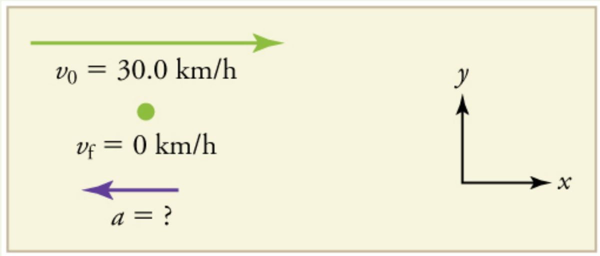


Figure 2.20

In this case, the train is decelerating and its acceleration is negative because it is toward the left. As in the previous example, we must find the change in velocity and the change in time and then solve for acceleration.

Solution

1. Identify the knowns. $v_0 = 30.0\text{ km/h}$, $v_f = 0\text{ km/h}$ (the train is stopped, so its velocity is 0), and $\Delta t = 8.00\text{ s}$.

2. Solve for the change in velocity, Δv.

$$\Delta v = v_f - v_0 = 0 - 30.0\text{ km/h} = -30.0\text{ km/h} \tag{2.16}$$

3. Plug in the knowns, Δv and Δt, and solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-30.0\text{ km/h}}{8.00\text{ s}} \tag{2.17}$$

4. Convert the units to meters and seconds.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \left(\frac{-30.0\text{ km/h}}{8.00\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = -1.04\text{ m/s}^2. \tag{2.18}$$

Discussion

The minus sign indicates that acceleration is to the left. This sign is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would oppose the motion. Again, acceleration is in the same direction as the *change* in velocity, which is negative here. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

The graphs of position, velocity, and acceleration vs. time for the trains in **Example 2.4** and **Example 2.5** are displayed in **Figure 2.21**. (We have taken the velocity to remain constant from 20 to 40 s, after which the train decelerates.)

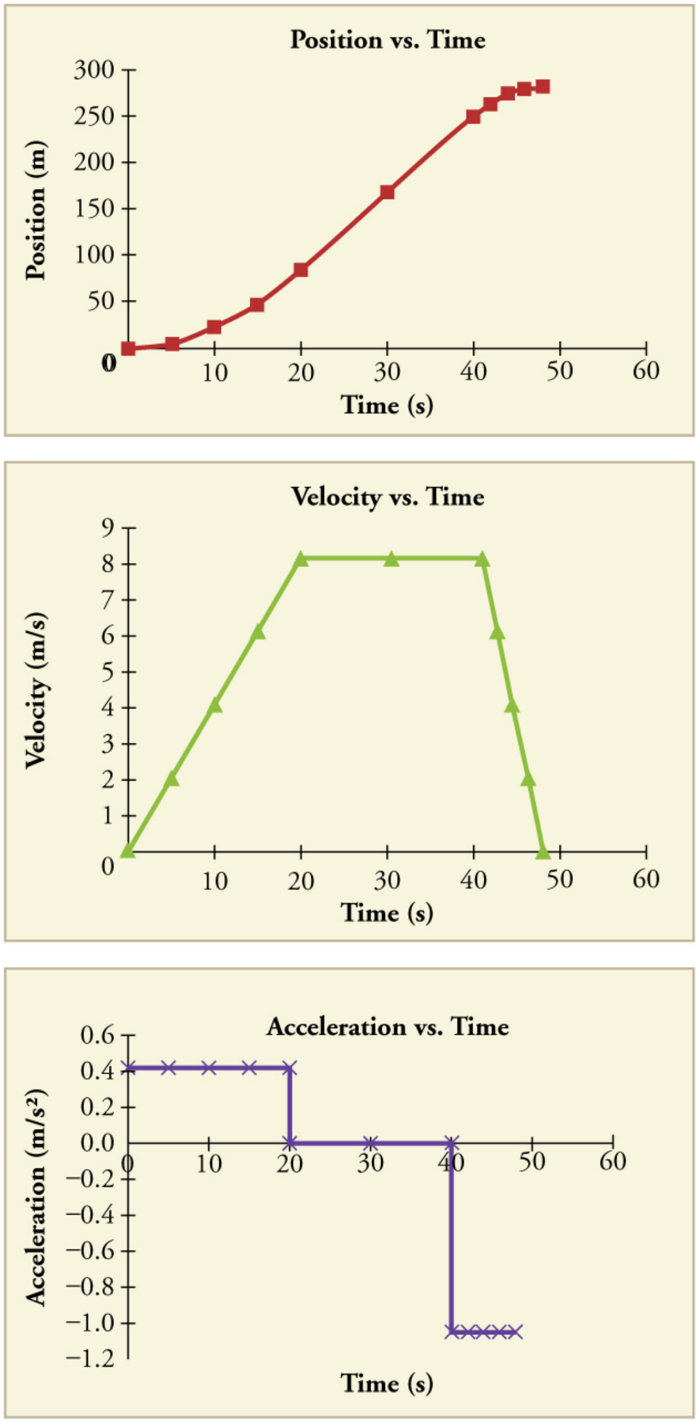


Figure 2.21 (a) Position of the train over time. Notice that the train's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate. (b) Velocity of the train over time. The train's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the train decelerates at the end of the journey. (c) The acceleration of the train over time. The train has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.

Example 2.6 Calculating Average Velocity: The Subway Train

What is the average velocity of the train in part b of **Example 2.2**, and shown again below, if it takes 5.00 min to make its trip?

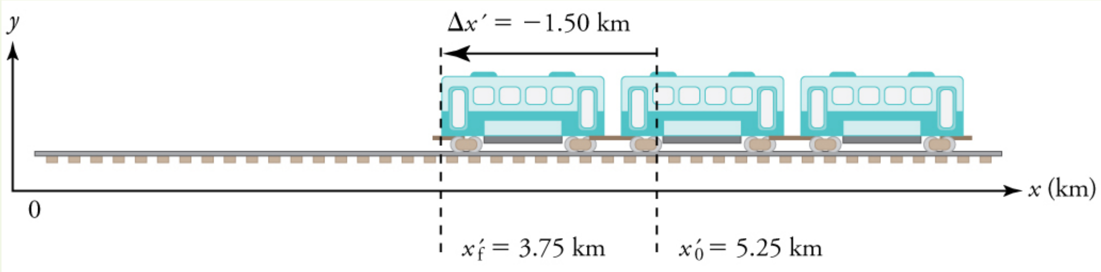


Figure 2.22

Strategy

Average velocity is displacement divided by time. It will be negative here, since the train moves to the left and has a negative displacement.

Solution

1. Identify the knowns. $x'_{\text{f}} = 3.75\text{ km}$, $x'_0 = 5.25\text{ km}$, $\Delta t = 5.00\text{ min}$.

2. Determine displacement, $\Delta x'$. We found $\Delta x'$ to be -1.5 km in **Example 2.2**.

3. Solve for average velocity.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \frac{-1.50\text{ km}}{5.00\text{ min}} \tag{2.19}$$

4. Convert units.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \left(\frac{-1.50\text{ km}}{5.00\text{ min}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right) = -18.0\text{ km/h} \tag{2.20}$$

Discussion

The negative velocity indicates motion to the left.

Example 2.7 Calculating Deceleration: The Subway Train

Finally, suppose the train in **Figure 2.22** slows to a stop from a velocity of 20.0 km/h in 10.0 s. What is its average acceleration?

Strategy

Once again, let's draw a sketch:

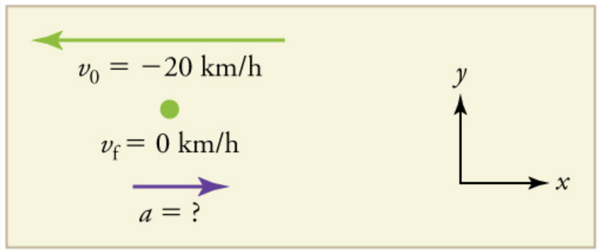


Figure 2.23

As before, we must find the change in velocity and the change in time to calculate average acceleration.

Solution

1. Identify the knowns. $v_0 = -20\text{ km/h}$, $v_{\text{f}} = 0\text{ km/h}$, $\Delta t = 10.0\text{ s}$.

2. Calculate Δv . The change in velocity here is actually positive, since

$$\Delta v = v_{\text{f}} - v_0 = 0 - (-20\text{ km/h}) = +20\text{ km/h}. \tag{2.21}$$

3. Solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+20.0\text{ km/h}}{10.0\text{ s}} \tag{2.22}$$

4. Convert units.

$$\bar{a} = \left(\frac{+20.0\text{ km/h}}{10.0\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = +0.556\text{ m/s}^2 \tag{2.23}$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train initially has a negative velocity (to the left) in this problem and a positive acceleration opposes the motion (and so it is to the right). Again, acceleration is in the same direction as the *change* in velocity, which is positive here. As in **Example 2.5**, this acceleration can be called a deceleration since it is in the direction opposite to the velocity.

Sign and Direction

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative acceleration as the slowing of an object. This was not the case in **Example 2.7**, where a positive acceleration slowed a negative velocity. The crucial distinction was that the acceleration was in the opposite direction from the velocity. In fact, a negative acceleration will *increase* a negative velocity. For example, the train moving to the left in **Figure 2.22** is sped up by an acceleration to the left. In that case, both v and a are negative. The plus and minus signs give the directions of the accelerations. If acceleration has the same sign as the change in velocity, the object is speeding up. If acceleration has the opposite sign of the change in velocity, the object is slowing down.

Check Your Understanding

An airplane lands on a runway traveling east. Describe its acceleration.

Solution
If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also decelerating: its acceleration is opposite in direction to its velocity.

PhET Explorations: Moving Man Simulation

Learn about position, velocity, and acceleration graphs. Move the little man back and forth with the mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you.

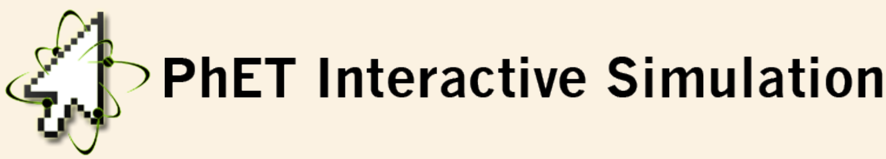


Figure 2.24 Moving Man (http://cnx.org/content/m42100/1.3/moving-man_en.jar)

2.5 Motion Equations for Constant Acceleration in One Dimension

Figure 2.25 Kinematic equations can help us describe and predict the motion of moving objects such as these kayaks racing in Newbury, England. (credit: Barry Skeates, Flickr)

We might know that the greater the acceleration of, say, a car moving away from a stop sign, the greater the displacement in a given time. But we have not developed a specific equation that relates acceleration and displacement. In this section, we develop some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration already covered.

Notation: *t*, *x*, *v*, *a*

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is $\Delta t = t_\text{f} - t_0$, taking $t_0 = 0$ means that $\Delta t = t_\text{f}$, the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is, x_0 *is the initial position* and v_0 *is the initial velocity*. We put no subscripts on the final values. That is, t *is the final time*, x *is the final position*, and v *is the final velocity*. This gives a simpler expression for elapsed time—now, $\Delta t = t$. It also simplifies the expression for displacement, which is now $\Delta x = x - x_0$. Also, it simplifies the expression for change in velocity, which is now $\Delta v = v - v_0$. To summarize, using the simplified notation, with the initial time taken to be zero,

$$\left.\begin{array}{lcl}\Delta t &=& t \\ \Delta x &=& x - x_0 \\ \Delta v &=& v - v_0\end{array}\right\} \tag{2.24}$$

where *the subscript 0 denotes an initial value and the absence of a subscript denotes a final value* in whatever motion is under consideration.

We now make the important assumption that *acceleration is constant*. This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal. That is,

$$\bar{a} = a = \text{constant}, \tag{2.25}$$

so we use the symbol a for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor degrade the accuracy of our treatment. For one thing, acceleration *is* constant in a great number of situations. Furthermore, in many other situations we can accurately describe motion by assuming a constant acceleration equal to the average acceleration for that motion. Finally, in motions where acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, the motion can be considered in separate parts, each of which has its own constant acceleration.

Solving for Displacement (Δx) and Final Position (x) from Average Velocity when Acceleration (a) is Constant

To get our first two new equations, we start with the definition of average velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t}. \tag{2.26}$$

Substituting the simplified notation for Δx and Δt yields

$$\bar{v} = \frac{x - x_0}{t}. \tag{2.27}$$

Solving for x yields

$$x = x_0 + \bar{v}\,t, \tag{2.28}$$

where the average velocity is

$$\bar{v} = \frac{v_0 + v}{2} \;(\text{constant } a). \tag{2.29}$$

The equation $\bar{v} = \frac{v_0 + v}{2}$ reflects the fact that, when acceleration is constant, v is just the simple average of the initial and final velocities. For example, if you steadily increase your velocity (that is, with constant acceleration) from 30 to 60 km/h, then your average velocity during this steady increase is 45 km/h. Using the equation $\bar{v} = \frac{v_0 + v}{2}$ to check this, we see that

$$\bar{v} = \frac{v_0 + v}{2} = \frac{30\text{ km/h} + 60\text{ km/h}}{2} = 45\text{ km/h}, \tag{2.30}$$

which seems logical.

Example 2.8 Calculating Displacement: How Far does the Jogger Run?

A jogger runs down a straight stretch of road with an average velocity of 4.00 m/s for 2.00 min. What is his final position, taking his initial position to be zero?

Strategy

Draw a sketch.

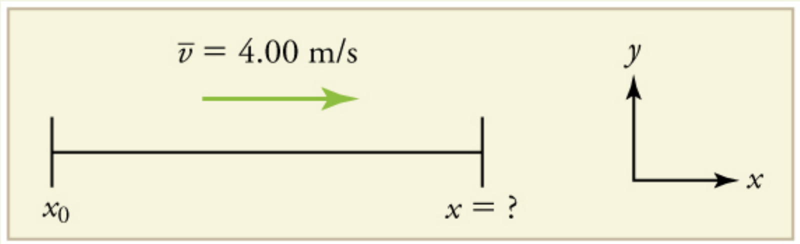


Figure 2.26

The final position x is given by the equation

$$x = x_0 + \bar{v}\,t. \tag{2.31}$$

To find x, we identify the values of x_0, $\bar{v}$, and t from the statement of the problem and substitute them into the equation.

Solution

1. Identify the knowns. $\bar{v} = 4.00\text{ m/s}$, $\Delta t = 2.00\text{ min}$, and $x_0 = 0\text{ m}$.

2. Enter the known values into the equation.

$$x = x_0 + \bar{v}t = 0 + (4.00\text{ m/s})(120\text{ s}) = 480\text{ m} \tag{2.32}$$

Discussion

Velocity and final displacement are both positive, which means they are in the same direction.

The equation $x = x_0 + \bar{v}t$ gives insight into the relationship between displacement, average velocity, and time. It shows, for example, that displacement is a linear function of average velocity. (By linear function, we mean that displacement depends on $\bar{v}$ rather than on $\bar{v}$ raised to some other power, such as $\bar{v}^2$. When graphed, linear functions look like straight lines with a constant slope.) On a car trip, for example, we will get twice as far in a given time if we average 90 km/h than if we average 45 km/h.

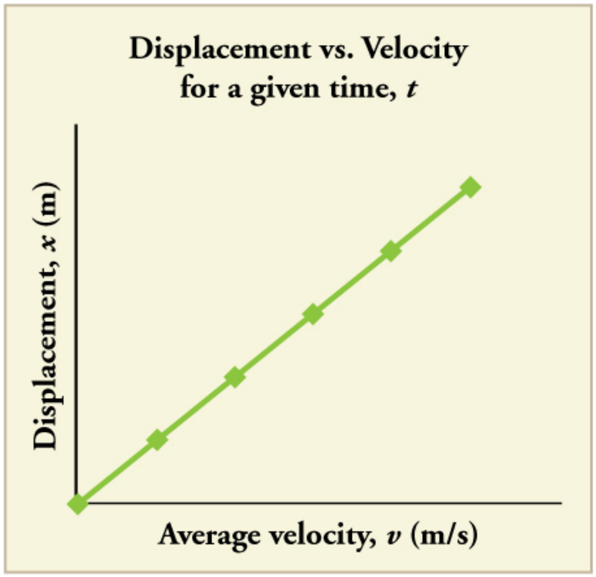


Figure 2.27 There is a linear relationship between displacement and average velocity. For a given time t, an object moving twice as fast as another object will move twice as far as the other object.

Solving for Final Velocity

We can derive another useful equation by manipulating the definition of acceleration.

$$a = \frac{\Delta v}{\Delta t} \tag{2.33}$$

Substituting the simplified notation for Δv and Δt gives us

$$a = \frac{v - v_0}{t}\text{ (constant }a\text{).} \tag{2.34}$$

Solving for v yields

$$v = v_0 + at\text{ (constant }a\text{).} \tag{2.35}$$

Example 2.9 Calculating Final Velocity: An Airplane Slowing Down after Landing

An airplane lands with an initial velocity of 70.0 m/s and then decelerates at 1.50 m/s^2 for 40.0 s. What is its final velocity?

Strategy

Draw a sketch. We draw the acceleration vector in the direction opposite the velocity vector because the plane is decelerating.

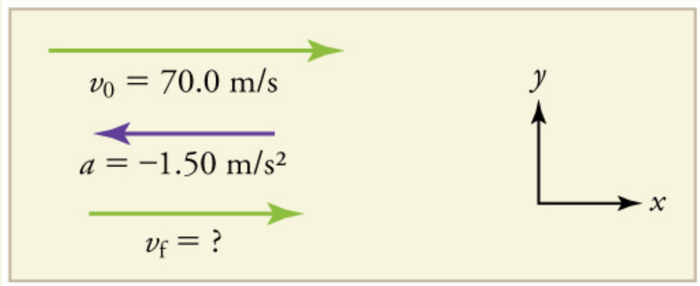


Figure 2.28

Solution

1. Identify the knowns. $\Delta v = 70.0\text{ m/s}$, $a = -1.50\text{ m/s}^2$, $t = 40.0\text{ s}$.

2. Identify the unknown. In this case, it is final velocity, v_f .

3. Determine which equation to use. We can calculate the final velocity using the equation $v = v_0 + at$.

4. Plug in the known values and solve.

$$v = v_0 + at = 70.0\text{ m/s} + \left(-1.50\text{ m/s}^2\right)(40.0\text{ s}) = 10.0\text{ m/s} \tag{2.36}$$

Discussion

The final velocity is much less than the initial velocity, as desired when slowing down, but still positive. With jet engines, reverse thrust could be maintained long enough to stop the plane and start moving it backward. That would be indicated by a negative final velocity, which is not the case here.

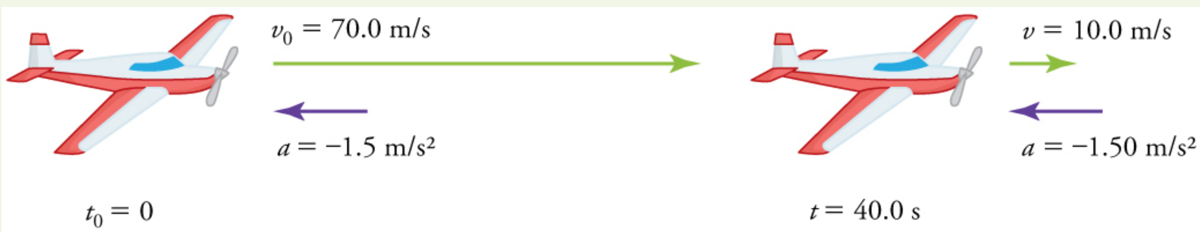


Figure 2.29 The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal. Note that the acceleration is negative because its direction is opposite to its velocity, which is positive.

In addition to being useful in problem solving, the equation $v = v_0 + at$ gives us insight into the relationships among velocity, acceleration, and time. From it we can see, for example, that

- final velocity depends on how large the acceleration is and how long it lasts
- if the acceleration is zero, then the final velocity equals the initial velocity $(v = v_0)$, as expected (i.e., velocity is constant)
- if a is negative, then the final velocity is less than the initial velocity

(All of these observations fit our intuition, and it is always useful to examine basic equations in light of our intuition and experiences to check that they do indeed describe nature accurately.)

Making Connections: Real-World Connection

Figure 2.30 The Space Shuttle *Endeavor* blasts off from the Kennedy Space Center in February 2010. (credit: Matthew Simantov, Flickr)

An intercontinental ballistic missile (ICBM) has a larger average acceleration than the Space Shuttle and achieves a greater velocity in the first minute or two of flight (actual ICBM burn times are classified—short-burn-time missiles are more difficult for an enemy to destroy). But the Space

Shuttle obtains a greater final velocity, so that it can orbit the earth rather than come directly back down as an ICBM does. The Space Shuttle does this by accelerating for a longer time.

Solving for Final Position When Velocity is Not Constant ($a \neq 0$)

We can combine the equations above to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with

$$v = v_0 + at. \tag{2.37}$$

Adding v_0 to each side of this equation and dividing by 2 gives

$$\frac{v_0 + v}{2} = v_0 + \frac{1}{2}at. \tag{2.38}$$

Since $\frac{v_0 + v}{2} = \bar{v}$ for constant acceleration, then

$$\bar{v} = v_0 + \frac{1}{2}at. \tag{2.39}$$

Now we substitute this expression for $\bar{v}$ into the equation for displacement, $x = x_0 + \bar{v}t$, yielding

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \text{ (constant } a\text{)}. \tag{2.40}$$

Example 2.10 Calculating Displacement of an Accelerating Object: Dragsters

Dragsters can achieve average accelerations of 26.0 m/s^2. Suppose such a dragster accelerates from rest at this rate for 5.56 s. How far does it travel in this time?

Figure 2.31 U.S. Army Top Fuel pilot Tony “The Sarge” Schumacher begins a race with a controlled burnout. (credit: Lt. Col. William Thurmond. Photo Courtesy of U.S. Army.)

Strategy

Draw a sketch.

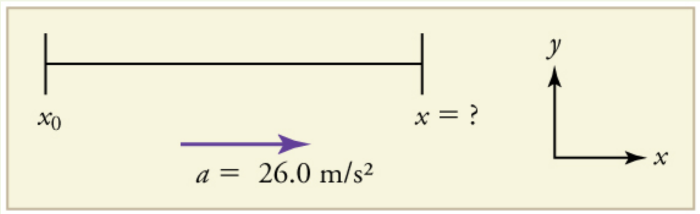


Figure 2.32

We are asked to find displacement, which is x if we take x_0 to be zero. (Think about it like the starting line of a race. It can be anywhere, but we call it 0 and measure all other positions relative to it.) We can use the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$ once we identify v_0, a, and t from the statement of the problem.

Solution

1. Identify the knowns. Starting from rest means that $v_0 = 0$, a is given as 26.0 m/s^2 and t is given as 5.56 s.

2. Plug the known values into the equation to solve for the unknown x:

$$x = x_0 + v_0 t + \frac{1}{2}at^2. \quad (2.41)$$

Since the initial position and velocity are both zero, this simplifies to

$$x = \frac{1}{2}at^2. \quad (2.42)$$

Substituting the identified values of a and t gives

$$x = \frac{1}{2}\left(26.0\text{ m/s}^2\right)(5.56\text{ s})^2, \quad (2.43)$$

yielding

$$x = 402\text{ m}. \quad (2.44)$$

Discussion

If we convert 402 m to miles, we find that the distance covered is very close to one quarter of a mile, the standard distance for drag racing. So the answer is reasonable. This is an impressive displacement in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this.

What else can we learn by examining the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$? We see that:

- displacement depends on the square of the elapsed time when acceleration is not zero. In **Example 2.10**, the dragster covers only one fourth of the total distance in the first half of the elapsed time
- if acceleration is zero, then the initial velocity equals average velocity ($v_0 = \bar{v}$) and $x = x_0 + v_0 t + \frac{1}{2}at^2$ becomes $x = x_0 + v_0 t$

Solving for Final Velocity when Velocity Is Not Constant ($a \neq 0$)

A fourth useful equation can be obtained from another algebraic manipulation of previous equations.

If we solve $v = v_0 + at$ for t, we get

$$t = \frac{v - v_0}{a}. \quad (2.45)$$

Substituting this and $\bar{v} = \frac{v_0 + v}{2}$ into $x = x_0 + \bar{v}t$, we get

$$v^2 = v_0^2 + 2a(x - x_0)\ (\text{constant}a). \quad (2.46)$$

Example 2.11 Calculating Final Velocity: Dragsters

Calculate the final velocity of the dragster in **Example 2.10** without using information about time.

Strategy

Draw a sketch.

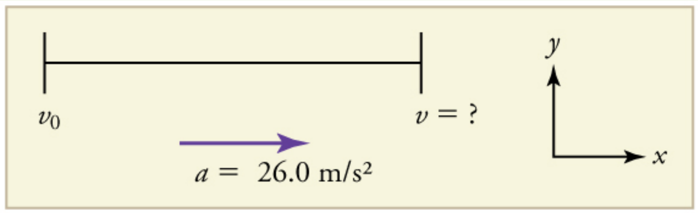


Figure 2.33

The equation $v^2 = v_0^2 + 2a(x - x_0)$ is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

Solution

1. Identify the known values. We know that $v_0 = 0$, since the dragster starts from rest. Then we note that $x - x_0 = 402\text{ m}$ (this was the answer in **Example 2.10**). Finally, the average acceleration was given to be $a = 26.0\text{ m/s}^2$.

2. Plug the knowns into the equation $v^2 = v_0^2 + 2a(x - x_0)$ and solve for v.

$$v^2 = 0 + 2\left(26.0\text{ m/s}^2\right)(402\text{ m}). \tag{2.47}$$

Thus

$$v^2 = 2.09\times10^4\text{ m}^2/\text{s}^2. \tag{2.48}$$

To get v , we take the square root:

$$v = \sqrt{2.09\times10^4\text{ m}^2/\text{s}^2} = 145\text{ m/s}. \tag{2.49}$$

Discussion

145 m/s is about 522 km/h or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation $v^2 = v_0^2 + 2a(x - x_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts
- For a fixed deceleration, a car that is going twice as fast doesn't simply stop in twice the distance—it takes much further to stop. (This is why we have reduced speed zones near schools.)

Putting Equations Together

In the following examples, we further explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The box below provides easy reference to the equations needed.

Summary of Kinematic Equations (constant a)

$$x = x_0 + \bar{v}t \tag{2.50}$$

$$\bar{v} = \frac{v_0 + v}{2} \tag{2.51}$$

$$v = v_0 + at \tag{2.52}$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \tag{2.53}$$

$$v^2 = v_0^2 + 2a(x - x_0) \tag{2.54}$$

Example 2.12 Calculating Displacement: How Far Does a Car Go When Coming to a Halt?

On dry concrete, a car can decelerate at a rate of 7.00 m/s^2 , whereas on wet concrete it can decelerate at only 5.00 m/s^2 . Find the distances necessary to stop a car moving at 30.0 m/s (about 110 km/h) (a) on dry concrete and (b) on wet concrete. (c) Repeat both calculations, finding the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of 0.500 s to get his foot on the brake.

Strategy

Draw a sketch.

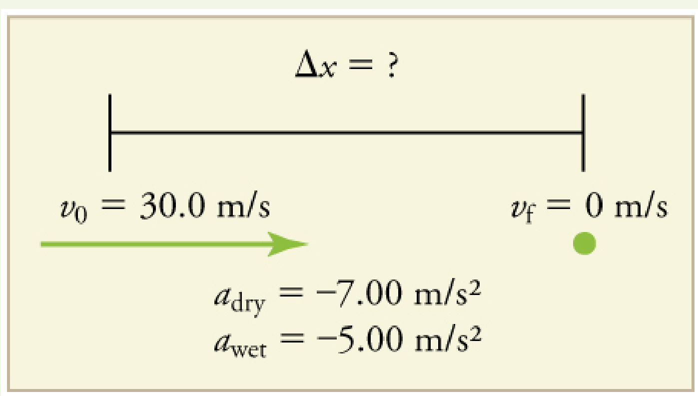


Figure 2.34

In order to determine which equations are best to use, we need to list all of the known values and identify exactly what we need to solve for. We shall do this explicitly in the next several examples, using tables to set them off.

Solution for (a)

1. Identify the knowns and what we want to solve for. We know that $v_0 = 30.0\text{ m/s}$; $v = 0$; $a = -7.00\text{ m/s}^2$ (a is negative because it is in a direction opposite to velocity). We take x_0 to be 0. We are looking for displacement Δx , or $x - x_0$.

2. Identify the equation that will help up solve the problem. The best equation to use is

$$v^2 = v_0^2 + 2a(x - x_0). \tag{2.55}$$

This equation is best because it includes only one unknown, x . We know the values of all the other variables in this equation. (There are other equations that would allow us to solve for x , but they require us to know the stopping time, t , which we do not know. We could use them but it would entail additional calculations.)

3. Rearrange the equation to solve for x .

$$x - x_0 = \frac{v^2 - v_0^2}{2a} \tag{2.56}$$

4. Enter known values.

$$x - 0 = \frac{0^2 - (30.0\text{ m/s})^2}{2\left(-7.00\text{ m/s}^2\right)} \tag{2.57}$$

Thus,

$$x = 64.3\text{ m on dry concrete.} \tag{2.58}$$

Solution for (b)

This part can be solved in exactly the same manner as Part A. The only difference is that the deceleration is -5.00 m/s^2 . The result is

$$x_{\text{wet}} = 90.0\text{ m on wet concrete.} \tag{2.59}$$

Solution for (c)

Once the driver reacts, the stopping distance is the same as it is in Parts A and B for dry and wet concrete. So to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume that the velocity remains constant during the driver's reaction time.

1. Identify the knowns and what we want to solve for. We know that $\bar{v} = 30.0\text{ m/s}$; $t_{\text{reaction}} = 0.500\text{ s}$; $a_{\text{reaction}} = 0$. We take $x_{0-\text{reaction}}$ to be 0. We are looking for x_{reaction} .

2. Identify the best equation to use.

$x = x_0 + \bar{v}t$ works well because the only unknown value is x , which is what we want to solve for.

3. Plug in the knowns to solve the equation.

$$x = 0 + (30.0\text{ m/s})(0.500\text{ s}) = 15.0\text{ m}. \tag{2.60}$$

This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly.

4. Add the displacement during the reaction time to the displacement when braking.

$$x_{\text{braking}} + x_{\text{reaction}} = x_{\text{total}} \tag{2.61}$$

a. 64.3 m + 15.0 m = 79.3 m when dry
b. 90.0 m + 15.0 m = 105 m when wet

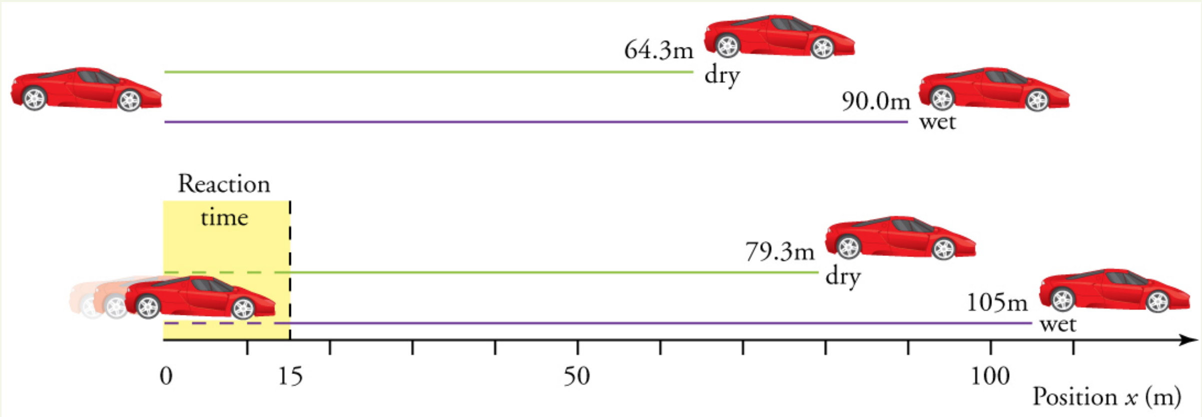


Figure 2.35 The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car initially traveling at 30.0 m/s. Also shown are the total distances traveled from the point where the driver first sees a light turn red, assuming a 0.500 s reaction time.

Discussion

The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet rather than dry pavement. It is interesting that reaction time adds significantly to the displacements. But more important is the general approach to solving problems. We identify the knowns and the quantities to be determined and then find an appropriate equation. There is often more than one way to solve a problem. The various parts of this example can in fact be solved by other methods, but the solutions presented above are the shortest.

Example 2.13 Calculating Time: A Car Merges into Traffic

Suppose a car merges into freeway traffic on a 200-m-long ramp. If its initial velocity is 10.0 m/s and it accelerates at 2.00 m/s^2, how long does it take to travel the 200 m up the ramp? (Such information might be useful to a traffic engineer.)

Strategy

Draw a sketch.

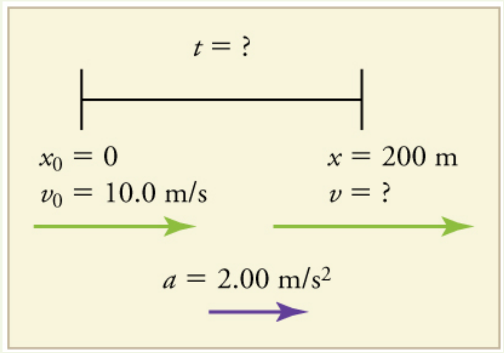


Figure 2.36

We are asked to solve for the time t. As before, we identify the known quantities in order to choose a convenient physical relationship (that is, an equation with one unknown, t).

Solution

1. Identify the knowns and what we want to solve for. We know that $v_0 = 10\text{ m/s}$; $a = 2.00\text{ m/s}^2$; and $x = 200\text{ m}$.

2. We need to solve for t. Choose the best equation. $x = x_0 + v_0 t + \frac{1}{2}at^2$ works best because the only unknown in the equation is the variable t for which we need to solve.

3. We will need to rearrange the equation to solve for t. In this case, it will be easier to plug in the knowns first.

$$200\text{ m} = 0\text{ m} + (10.0\text{ m/s})t + \frac{1}{2}\left(2.00\text{ m/s}^2\right)t^2 \tag{2.62}$$

4. Simplify the equation. The units of meters (m) cancel because they are in each term. We can get the units of seconds (s) to cancel by taking $t = t\text{ s}$, where t is the magnitude of time and s is the unit. Doing so leaves

$$200 = 10t + t^2. \tag{2.63}$$

5. Use the quadratic formula to solve for t.

(a) Rearrange the equation to get 0 on one side of the equation.

$$t^2 + 10t - 200 = 0 \tag{2.64}$$

This is a quadratic equation of the form

$$at^2 + bt + c = 0, \tag{2.65}$$

where the constants are $a = 1.00, b = 10.0, \text{ and } c = -200$.

(b) Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{2.66}$$

This yields two solutions for t, which are

$$t = 10.0 \text{ and} -20.0. \tag{2.67}$$

In this case, then, the time is $t = t$ in seconds, or

$$t = 10.0\text{ s and } -20.0\text{ s}. \tag{2.68}$$

A negative value for time is unreasonable, since it would mean that the event happened 20 s before the motion began. We can discard that solution. Thus,

$$t = 10.0\text{ s.} \tag{2.69}$$

Discussion

Whenever an equation contains an unknown squared, there will be two solutions. In some problems both solutions are meaningful, but in others, such as the above, only one solution is reasonable. The 10.0 s answer seems reasonable for a typical freeway on-ramp.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships. **Problem-Solving Basics** discusses problem-solving basics and outlines an approach that will help you succeed in this invaluable task.

Making Connections: Take-Home Experiment—Breaking News

We have been using SI units of meters per second squared to describe some examples of acceleration or deceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking deceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration, $\bar{a} = \Delta v / \Delta t$. While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the deceleration in miles per hour per second. Convert this to meters per second squared and compare with other decelerations mentioned in this chapter. Calculate the distance traveled in braking.

Check Your Understanding

A manned rocket accelerates at a rate of 20 m/s^2 during launch. How long does it take the rocket reach a velocity of 400 m/s?

Solution

To answer this, choose an equation that allows you to solve for time t , given only a , v_0 , and v .

$$v = v_0 + at \tag{2.70}$$

Rearrange to solve for t .

$$t = \frac{v - v}{a} = \frac{400\text{ m/s} - 0\text{ m/s}}{20\text{ m/s}^2} = 20\text{ s} \tag{2.71}$$

2.6 Problem-Solving Basics for One-Dimensional Kinematics

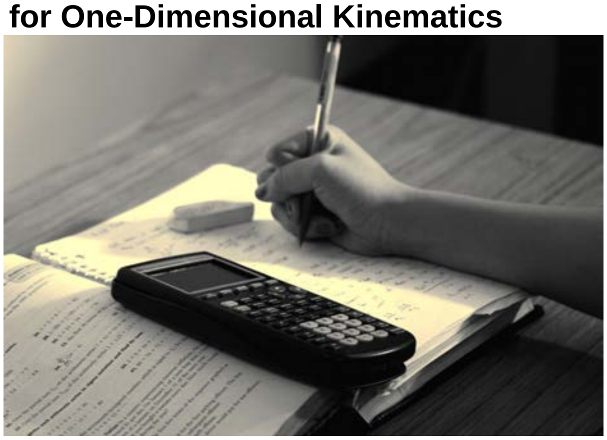


Figure 2.37 Problem-solving skills are essential to your success in Physics. (credit: scui3asteveo, Flickr)

Problem-solving skills are obviously essential to success in a quantitative course in physics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in this text and for applying physics in everyday and professional life.

Problem-Solving Steps

While there is no simple step-by-step method that works for every problem, the following general procedures facilitate problem solving and make it more meaningful. A certain amount of creativity and insight is required as well.

Step 1

Examine the situation to determine which physical principles are involved. It often helps to *draw a simple sketch* at the outset. You will also need to decide which direction is positive and note that on your sketch. Once you have identified the physical principles, it is much easier to find and apply the equations representing those principles. Although finding the correct equation is essential, keep in mind that equations represent physical principles, laws of nature, and relationships among physical quantities. Without a conceptual understanding of a problem, a numerical solution is meaningless.

Step 2

Make a list of what is given or can be inferred from the problem as stated (identify the knowns). Many problems are stated very succinctly and require some inspection to determine what is known. A sketch can also be very useful at this point. Formally identifying the knowns is of particular importance in applying physics to real-world situations. Remember, “stopped” means velocity is zero, and we often can take initial time and position as zero.

Step 3

Identify exactly what needs to be determined in the problem (identify the unknowns). In complex problems, especially, it is not always obvious what needs to be found or in what sequence. Making a list can help.

Step 4

Find an equation or set of equations that can help you solve the problem. Your list of knowns and unknowns can help here. It is easiest if you can find equations that contain only one unknown—that is, all of the other variables are known, so you can easily solve for the unknown. If the equation contains more than one unknown, then an additional equation is needed to solve the problem. In some problems, several unknowns must be determined to get at the one needed most. In such problems it is especially important to keep physical principles in mind to avoid going astray in a sea of equations. You may have to use two (or more) different equations to get the final answer.

Step 5

Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units. This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made. However, be warned that correct units do not guarantee that the numerical part of the answer is also correct.

Step 6

Check the answer to see if it is reasonable: Does it make sense? This final step is extremely important—the goal of physics is to accurately describe nature. To see if the answer is reasonable, check both its magnitude and its sign, in addition to its units. Your judgment will improve as you solve more and more physics problems, and it will become possible for you to make finer and finer judgments regarding whether nature is adequately described by the answer to a problem. This step brings the problem back to its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than just being able to mechanically solve a problem.

When solving problems, we often perform these steps in different order, and we also tend to do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience, and the basics of problem solving become almost automatic. One way to get practice is to work out the text’s examples for yourself as you read. Another is to work as many end-of-section problems as possible, starting with the easiest to build confidence and progressing to the more difficult. Once you become involved in physics, you will see it all around you, and you can begin to apply it to situations you encounter outside the classroom, just as is done in many of the applications in this text.

Unreasonable Results

Physics must describe nature accurately. Some problems have results that are unreasonable because one premise is unreasonable or because certain premises are inconsistent with one another. The physical principle applied correctly then produces an unreasonable result. For example, if a person starting a foot race accelerates at $0.40\ \text{m/s}^2$ for 100 s, his final speed will be 40 m/s (about 150 km/h)—clearly unreasonable because the time of 100 s is an unreasonable premise. The physics is correct in a sense, but there is more to describing nature than just manipulating equations correctly. Checking the result of a problem to see if it is reasonable does more than help uncover errors in problem solving—it also builds intuition in judging whether nature is being accurately described.

Use the following strategies to determine whether an answer is reasonable and, if it is not, to determine what is the cause.

Step 1

Solve the problem using strategies as outlined and in the format followed in the worked examples in the text. In the example given in the preceding paragraph, you would identify the givens as the acceleration and time and use the equation below to find the unknown final velocity. That is,

$$v = v_0 + at = 0 + \left(0.40\ \text{m/s}^2\right)(100\ \text{s}) = 40\ \text{m/s}. \tag{2.72}$$

Step 2

Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, …? In this case, you may need to convert meters per second into a more familiar unit, such as miles per hour.

$$\left(\frac{40\ \text{m}}{\text{s}}\right)\left(\frac{3.28\ \text{ft}}{\text{m}}\right)\left(\frac{1\ \text{mi}}{5280\ \text{ft}}\right)\left(\frac{60\ \text{s}}{\text{min}}\right)\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = 89\ \text{mph} \tag{2.73}$$

This velocity is about four times greater than a person can run—so it is too large.

Step 3

If the answer is unreasonable, look for what specifically could cause the identified difficulty. In the example of the runner, there are only two assumptions that are suspect. The acceleration could be too great or the time too long. First look at the acceleration and think about what the number means. If someone accelerates at $0.40\ \text{m/s}^2$, their velocity is increasing by 0.4 m/s each second. Does this seem reasonable? If so, the time must be too long. It is not possible for someone to accelerate at a constant rate of $0.40\ \text{m/s}^2$ for 100 s (almost two minutes).

2.7 Falling Objects

Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration*, *independent of their mass*. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones.

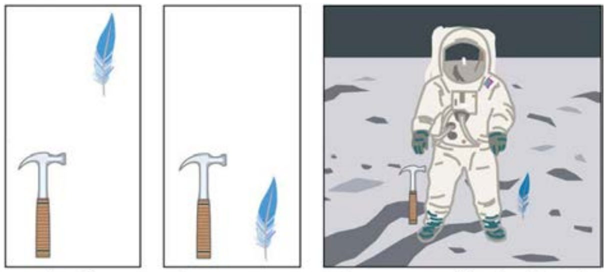


Figure 2.38 A hammer and a feather will fall with the same constant acceleration if air resistance is considered negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated on the Moon in 1971, where the acceleration due to gravity is only 1.67 m/s^2 .

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball will reach the ground after a hard baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, while friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them. For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in **free-fall**.

The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called the **acceleration due to gravity**. The acceleration due to gravity is *constant*, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations to us. The acceleration due to gravity is so important that its magnitude is given its own symbol, g . It is constant at any given location on Earth and has the average value

$$g = 9.80\text{ m/s}^2. \tag{2.74}$$

Although g varies from 9.78 m/s^2 to 9.83 m/s^2 , depending on latitude, altitude, underlying geological formations, and local topography, the average value of 9.80 m/s^2 will be used in this text unless otherwise specified. The direction of the acceleration due to gravity is *downward (towards the center of Earth)*. In fact, its direction *defines* what we call vertical. Note that whether the acceleration a in the kinematic equations has the value $+g$ or $-g$ depends on how we define our coordinate system. If we define the upward direction as positive, then $a = -g = -9.80\text{ m/s}^2$, and if we define the downward direction as positive, then $a = g = 9.80\text{ m/s}^2$.

One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So we start by considering straight up and down motion with no air resistance or friction. These assumptions mean that the velocity (if there is any) is vertical. If the object is dropped, we know the initial velocity is zero. Once the object has left contact with whatever held or threw it, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude g . We will also represent vertical displacement with the symbol y and use x for horizontal displacement.

Kinematic Equations for Objects in Free-Fall where Acceleration = *-g*

$$v = v_0 - gt \tag{2.75}$$

$$y = y_0 + v_0 t - \frac{1}{2}gt^2 \tag{2.76}$$

$$v^2 = v_0^2 - 2g(y - y_0) \tag{2.77}$$

Example 2.14 Calculating Position and Velocity of a Falling Object: A Rock Thrown Upward

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity of 13.0 m/s. The rock misses the edge of the cliff as it falls back to earth. Calculate the position and velocity of the rock 1.00 s, 2.00 s, and 3.00 s after it is thrown, neglecting the effects of air resistance.

Strategy

Draw a sketch.

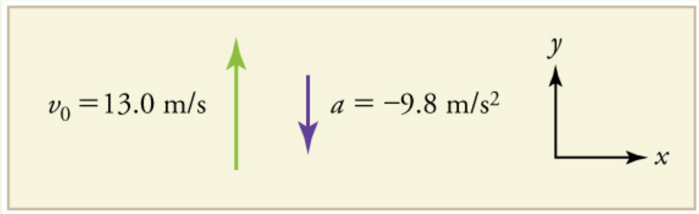


Figure 2.39

We are asked to determine the position y at various times. It is reasonable to take the initial position y_0 to be zero. This problem involves one-dimensional motion in the vertical direction. We use plus and minus signs to indicate direction, with up being positive and down negative. Since up is positive, and the rock is thrown upward, the initial velocity must be positive too. The acceleration due to gravity is downward, so a is negative. It is crucial that the initial velocity and the acceleration due to gravity have opposite signs. Opposite signs indicate that the acceleration due to gravity opposes the initial motion and will slow and eventually reverse it.

Since we are asked for values of position and velocity at three times, we will refer to these as y_1 and v_1; y_2 and v_2; and y_3 and v_3.

Solution for Position y_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0\text{ m/s}$; $a = -g = -9.80\text{ m/s}^2$; and $t = 1.00\text{ s}$.

2. Identify the best equation to use. We will use $y = y_0 + v_0 t + \frac{1}{2}at^2$ because it includes only one unknown, y (or y_1, here), which is the value we want to find.

3. Plug in the known values and solve for y_1.

$$y = 0 + (13.0\text{ m/s})(1.00\text{ s}) + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)(1.00\text{ s})^2 = 8.10\text{ m} \tag{2.78}$$

Discussion

The rock is 8.10 m above its starting point at $t = 1.00$ s, since $y_1 > y_0$. It could be *moving* up or down; the only way to tell is to calculate v_1 and find out if it is positive or negative.

Solution for Velocity v_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0\text{ m/s}$; $a = -g = -9.80\text{ m/s}^2$; and $t = 1.00\text{ s}$. We also know from the solution above that $y_1 = 8.10\text{ m}$.

2. Identify the best equation to use. The most straightforward is $v = v_0 - gt$ (from $v = v_0 + at$, where $a = \text{gravitational acceleration} = -g$).

3. Plug in the knowns and solve.

$$v_1 = v_0 - gt = 13.0\text{ m/s} - \left(9.80\text{ m/s}^2\right)(1.00\text{ s}) = 3.20\text{ m/s} \tag{2.79}$$

Discussion

The positive value for v_1 means that the rock is still heading upward at $t = 1.00\text{ s}$. However, it has slowed from its original 13.0 m/s, as expected.

Solution for Remaining Times

The procedures for calculating the position and velocity at $t = 2.00\text{ s}$ and 3.00 s are the same as those above. The results are summarized in **Table 2.1** and illustrated in **Figure 2.40**.

Table 2.1 Results

Time, *t*	Position, *y*	Velocity, *v*	Acceleration, *a*
1.00 s	8.10 m	3.20 m/s	-9.80 m/s^2
2.00 s	6.40 m	−6.60 m/s	-9.80 m/s^2
3.00 s	−5.10 m	−16.4 m/s	-9.80 m/s^2

Graphing the data helps us understand it more clearly.

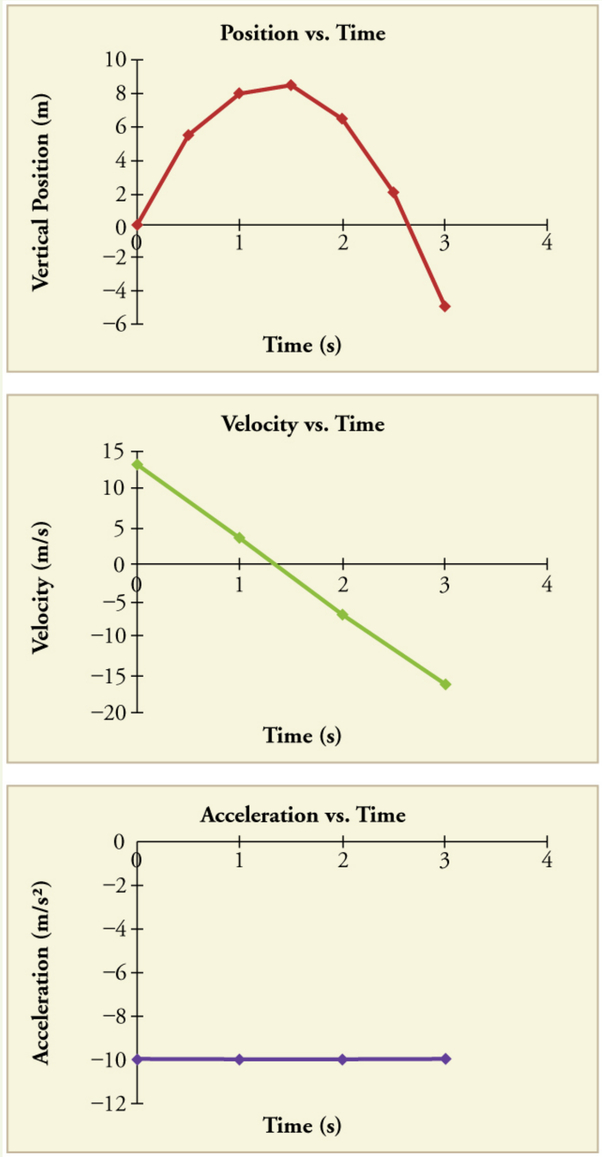


Figure 2.40 Vertical position, vertical velocity, and vertical acceleration vs. time for a rock thrown vertically up at the edge of a cliff. Notice that velocity changes linearly with time and that acceleration is constant. *Misconception Alert!* Notice that the position vs. time graph shows vertical position only. It is easy to get the impression that the graph shows some horizontal motion—the shape of the graph looks like the path of a projectile. But this is not the case; the horizontal axis is *time*, not space. The actual path of the rock in space is straight up, and straight down.

Discussion

The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since y_1 and v_1 are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both y_3 and v_3 are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.5 s), its velocity is zero, but its acceleration is still -9.80 m/s^2. Its acceleration is -9.80 m/s^2 for the whole trip—while it is moving up and while it is moving down. Note that the values for y are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.

Making Connections: Take-Home Experiment—Reaction Time

A simple experiment can be done to determine your reaction time. Have a friend hold a ruler between your thumb and index finger, separated by about 1 cm. Note the mark on the ruler that is right between your fingers. Have your friend drop the ruler unexpectedly, and try to catch it between your two fingers. Note the new reading on the ruler. Assuming acceleration is that due to gravity, calculate your reaction time. How far would you travel in a car (moving at 30 m/s) if the time it took your foot to go from the gas pedal to the brake was twice this reaction time?

Example 2.15 Calculating Velocity of a Falling Object: A Rock Thrown Down

What happens if the person on the cliff throws the rock straight down, instead of straight up? To explore this question, calculate the velocity of the rock when it is 5.10 m below the starting point, and has been thrown downward with an initial speed of 13.0 m/s.

Strategy

Draw a sketch.

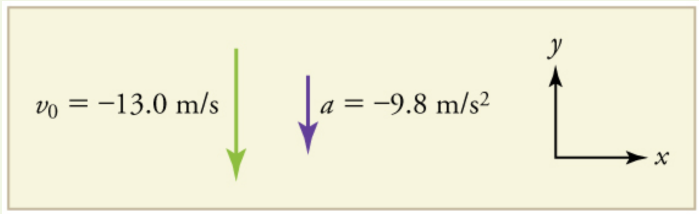


Figure 2.41

Since up is positive, the final position of the rock will be negative because it finishes below the starting point at $y_0 = 0$. Similarly, the initial velocity is downward and therefore negative, as is the acceleration due to gravity. We expect the final velocity to be negative since the rock will continue to move downward.

Solution

1. Identify the knowns. $y_0 = 0$; $y_1 = -5.10\text{ m}$; $v_0 = -13.0\text{ m/s}$; $a = -g = -9.80\text{ m/s}^2$.

2. Choose the kinematic equation that makes it easiest to solve the problem. The equation $v^2 = v_0^2 + 2a(y - y_0)$ works well because the only unknown in it is v. (We will plug y_1 in for y.)

3. Enter the known values

$$v^2 = (-13.0\text{ m/s})^2 + 2\left(-9.80\text{ m/s}^2\right)(-5.10\text{ m} - 0\text{ m}) = 268.96\text{ m}^2/\text{s}^2, \quad (2.80)$$

where we have retained extra significant figures because this is an intermediate result.

Taking the square root, and noting that a square root can be positive or negative, gives

$$v = \pm 16.4\text{ m/s}. \quad (2.81)$$

The negative root is chosen to indicate that the rock is still heading down. Thus,

$$v = -16.4\text{ m/s}. \quad (2.82)$$

Discussion

Note that *this is exactly the same velocity the rock had at this position when it was thrown straight upward with the same initial speed*. (See **Example 2.14** and **Figure 2.42**(a).) This is not a coincidental result. Because we only consider the acceleration due to gravity in this problem, the *speed* of a falling object depends only on its initial speed and its vertical position relative to the starting point. For example, if the velocity of the rock is calculated at a height of 8.10 m above the starting point (using the method from **Example 2.14**) when the initial velocity is 13.0 m/s straight up, a result of $\pm 3.20\text{ m/s}$ is obtained. Here both signs are meaningful; the positive value occurs when the rock is at 8.10 m and heading up, and the negative value occurs when the rock is at 8.10 m and heading back down. It has the same *speed* but the opposite direction.

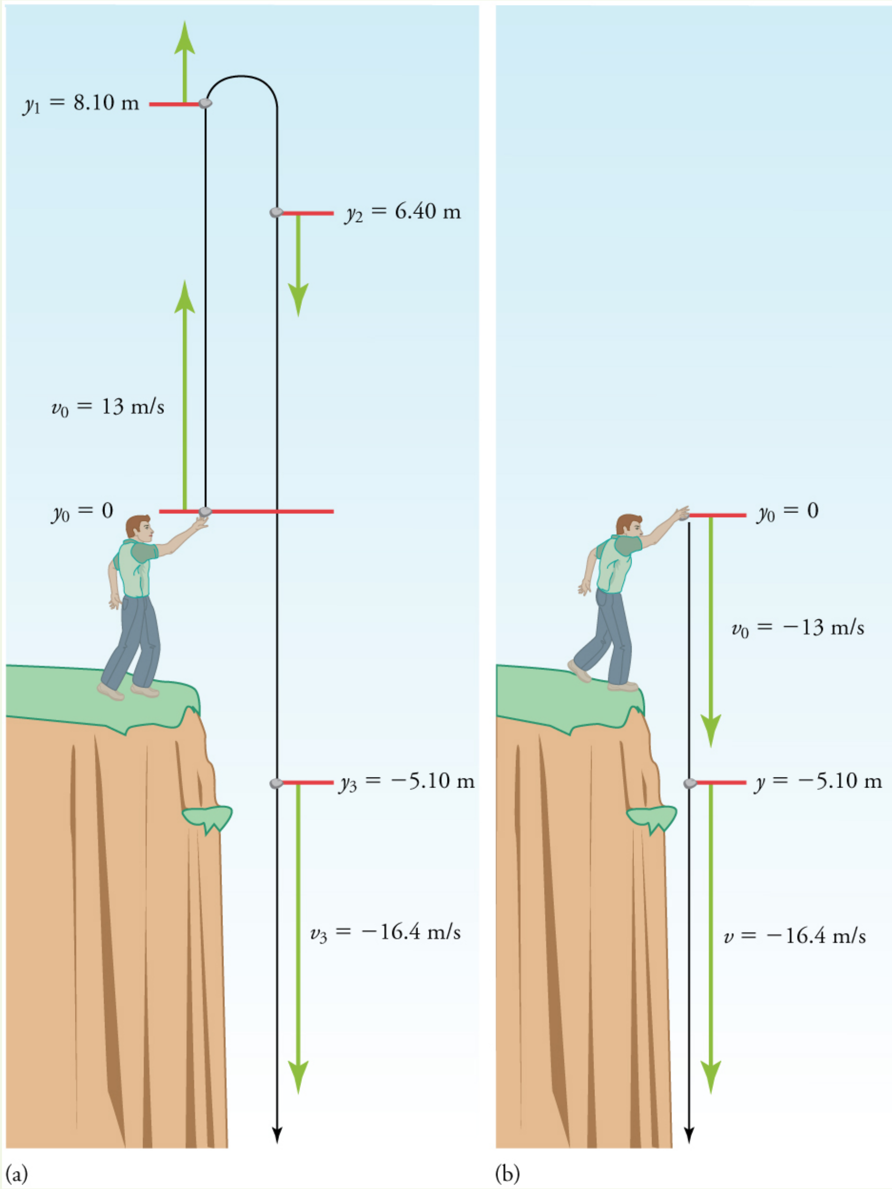
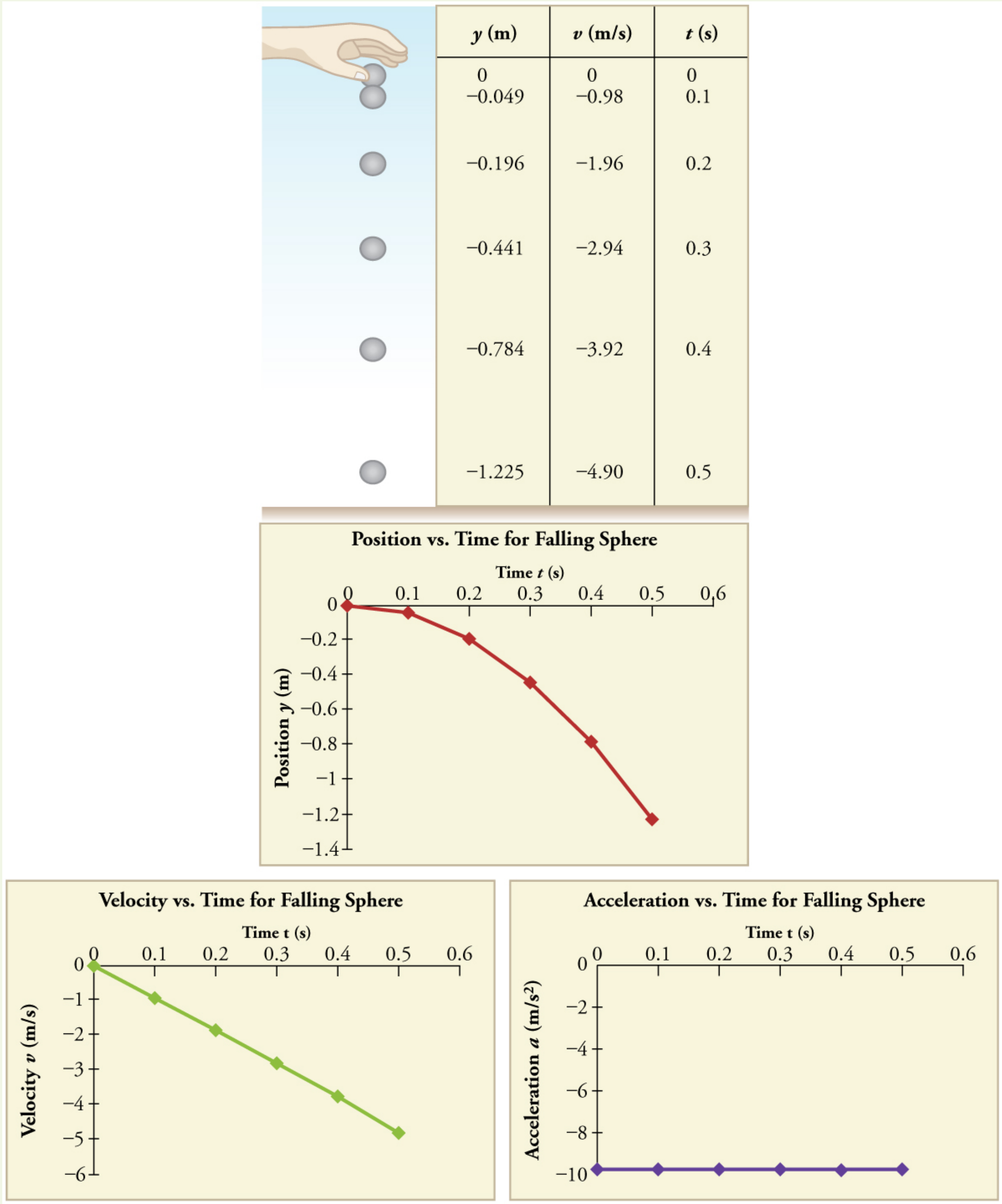


Figure 2.42 (a) A person throws a rock straight up, as explored in **Example 2.14**. The arrows are velocity vectors at 0, 1.00, 2.00, and 3.00 s. (b) A person throws a rock straight down from a cliff with the same initial speed as before, as in **Example 2.15**. Note that at the same distance below the point of release, the rock has the same velocity in both cases.

Another way to look at it is this: In **Example 2.14**, the rock is thrown up with an initial velocity of 13.0 m/s. It rises and then falls back down. When its position is $y = 0$ on its way back down, its velocity is -13.0 m/s. That is, it has the same speed on its way down as on its way up. We would then expect its velocity at a position of $y = -5.10\text{ m}$ to be the same whether we have thrown it upwards at $+13.0\text{ m/s}$ or thrown it downwards at -13.0 m/s. The velocity of the rock on its way down from $y = 0$ is the same whether we have thrown it up or down to start with, as long as the speed with which it was initially thrown is the same.

Example 2.16 Find *g* from Data on a Falling Object

The acceleration due to gravity on Earth differs slightly from place to place, depending on topography (e.g., whether you are on a hill or in a valley) and subsurface geology (whether there is dense rock like iron ore as opposed to light rock like salt beneath you.) The precise acceleration due to gravity can be calculated from data taken in an introductory physics laboratory course. An object, usually a metal ball for which air resistance is negligible, is dropped and the time it takes to fall a known distance is measured. See, for example, **Figure 2.43**. Very precise results can be produced with this method if sufficient care is taken in measuring the distance fallen and the elapsed time.



y (m)	v (m/s)	t (s)
0	0	0
−0.049	−0.98	0.1
−0.196	−1.96	0.2
−0.441	−2.94	0.3
−0.784	−3.92	0.4
−1.225	−4.90	0.5

Figure 2.43 Positions and velocities of a metal ball released from rest when air resistance is negligible. Velocity is seen to increase linearly with time while displacement increases with time squared. Acceleration is a constant and is equal to gravitational acceleration.

Suppose the ball falls 1.0000 m in 0.45173 s. Assuming the ball is not affected by air resistance, what is the precise acceleration due to gravity at this location?

Strategy

Draw a sketch.

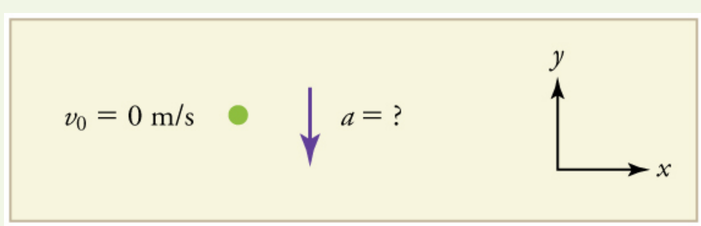


Figure 2.44

We need to solve for acceleration a. Note that in this case, displacement is downward and therefore negative, as is acceleration.

Solution

1. Identify the knowns. $y_0 = 0$; $y = –1.0000\text{ m}$; $t = 0.45173$; $v_0 = 0$.

2. Choose the equation that allows you to solve for a using the known values.

$$y = y_0 + v_0 t + \frac{1}{2}at^2 \tag{2.83}$$

3. Substitute 0 for v_0 and rearrange the equation to solve for a. Substituting 0 for v_0 yields

$$y = y_0 + \frac{1}{2}at^2. \tag{2.84}$$

Solving for a gives

$$a = \frac{2(y - y_0)}{t^2}. \tag{2.85}$$

4. Substitute known values yields

$$a = \frac{2(-1.0000\text{ m} - 0)}{(0.45173\text{ s})^2} = -9.8010\text{ m/s}^2, \tag{2.86}$$

so, because $a = -g$ with the directions we have chosen,

$$g = 9.8010\text{ m/s}^2. \tag{2.87}$$

Discussion

The negative value for a indicates that the gravitational acceleration is downward, as expected. We expect the value to be somewhere around the average value of 9.80 m/s^2, so 9.8010 m/s^2 makes sense. Since the data going into the calculation are relatively precise, this value for g is more precise than the average value of 9.80 m/s^2; it represents the local value for the acceleration due to gravity.

Check Your Understanding

A chunk of ice breaks off a glacier and falls 30.0 meters before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water?

Solution

We know that initial position $y_0 = 0$, final position $y = -30.0\text{ m}$, and $a = -g = -9.80\text{ m/s}^2$. We can then use the equation $y = y_0 + v_0 t + \frac{1}{2}at^2$ to solve for t. Inserting $a = -g$, we obtain

$$\begin{aligned} y &= 0 + 0 - \frac{1}{2}gt^2 \\ t^2 &= \frac{2y}{-g} \\ t &= \pm\sqrt{\frac{2y}{-g}} = \pm\sqrt{\frac{2(-30.0\text{ m})}{-9.80\text{ m/s}^2}} = \pm\sqrt{6.12\text{ s}^2} = 2.47\text{ s} \approx 2.5\text{ s} \end{aligned} \tag{2.88}$$

where we take the positive value as the physically relevant answer. Thus, it takes about 2.5 seconds for the piece of ice to hit the water.

PhET Explorations: Equation Grapher

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = bx$) to see how they add to generate the polynomial curve.

PhET Interactive Simulation

Figure 2.45 Equation Grapher (http://cnx.org/content/m42102/1.5/equation-grapher_en.jar)

2.8 Graphical Analysis of One-Dimensional Motion

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information; they also reveal relationships between physical quantities. This section uses graphs of displacement, velocity, and acceleration versus time to illustrate one-dimensional kinematics.

Slopes and General Relationships

First note that graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against one another in such a graph, the horizontal axis is usually considered to be an **independent variable** and the vertical axis a **dependent variable**. If we call the horizontal axis the x -axis and the vertical axis the y -axis, as in **Figure 2.46**, a straight-line graph has the general form

$$y = mx + b. \tag{2.89}$$

Here m is the **slope**, defined to be the rise divided by the run (as seen in the figure) of the straight line. The letter b is used for the ***y*-intercept**, which is the point at which the line crosses the vertical axis.

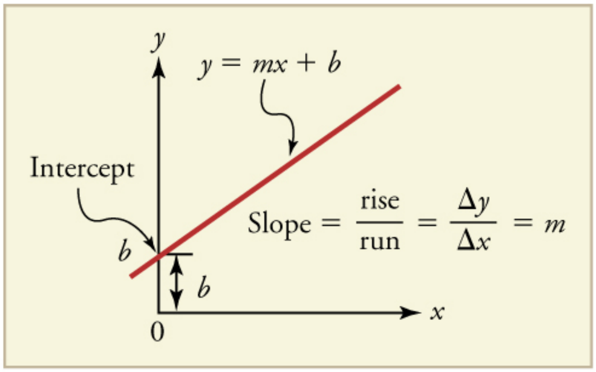


Figure 2.46 A straight-line graph. The equation for a straight line is $y = mx + b$.

Graph of Displacement vs. Time (*a* = 0, so *v* is constant)

Time is usually an independent variable that other quantities, such as displacement, depend upon. A graph of displacement versus time would, thus, have x on the vertical axis and t on the horizontal axis. **Figure 2.47** is just such a straight-line graph. It shows a graph of displacement versus time for a jet-powered car on a very flat dry lake bed in Nevada.

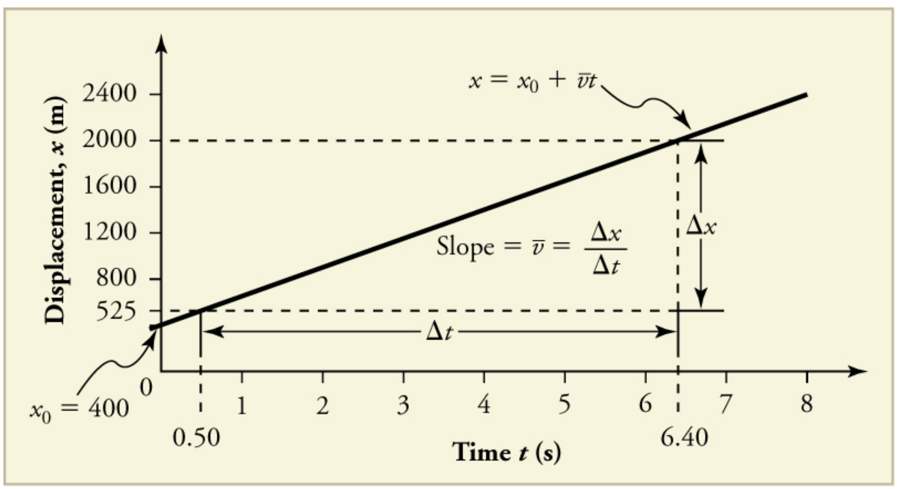


Figure 2.47 Graph of displacement versus time for a jet-powered car on the Bonneville Salt Flats.

Using the relationship between dependent and independent variables, we see that the slope in the graph above is average velocity $\bar{v}$ and the intercept is displacement at time zero—that is, x_0 . Substituting these symbols into $y = mx + b$ gives

$$x = \bar{v}t + x_0 \tag{2.90}$$

or

$$x = x_0 + \bar{v}t. \tag{2.91}$$

Thus a graph of displacement versus time gives a general relationship among displacement, velocity, and time, as well as giving detailed numerical information about a specific situation.

The Slope of *x* vs. *t*

The slope of the graph of displacement x vs. time t is velocity v .

$$\text{slope} = \frac{\Delta x}{\Delta t} = v \tag{2.92}$$

Notice that this equation is the same as that derived algebraically from other motion equations in **Motion Equations for Constant Acceleration in One Dimension**.

From the figure we can see that the car has a displacement of 400 m at time 0.650 m at t = 1.0 s, and so on. Its displacement at times other than those listed in the table can be read from the graph; furthermore, information about its velocity and acceleration can also be obtained from the graph.

Example 2.17 Determining Average Velocity from a Graph of Displacement versus Time: Jet Car

Find the average velocity of the car whose position is graphed in **Figure 2.47**.

Strategy

The slope of a graph of x vs. t is average velocity, since slope equals rise over run. In this case, rise = change in displacement and run = change in time, so that

$$\text{slope} = \frac{\Delta x}{\Delta t} = \bar{v}. \tag{2.93}$$

Since the slope is constant here, any two points on the graph can be used to find the slope. (Generally speaking, it is most accurate to use two widely separated points on the straight line. This is because any error in reading data from the graph is proportionally smaller if the interval is larger.)

Solution

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)

2. Substitute the x and t values of the chosen points into the equation. Remember in calculating change (Δ) we always use final value minus initial value.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2000\text{ m} - 525\text{ m}}{6.4\text{ s} - 0.50\text{ s}}, \tag{2.94}$$

yielding

$$\bar{v} = 250\text{ m/s}. \tag{2.95}$$

Discussion

This is an impressively large land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 60 mi/h (27 m/s or 96 km/h), but considerably shy of the record of 343 m/s (1234 km/h or 766 mi/h) set in 1997.

Graphs of Motion when a is constant but $a \neq 0$

The graphs in **Figure 2.48** below represent the motion of the jet-powered car as it accelerates toward its top speed, but only during the time when its acceleration is constant. Time starts at zero for this motion (as if measured with a stopwatch), and the displacement and velocity are initially 200 m and 15 m/s, respectively.

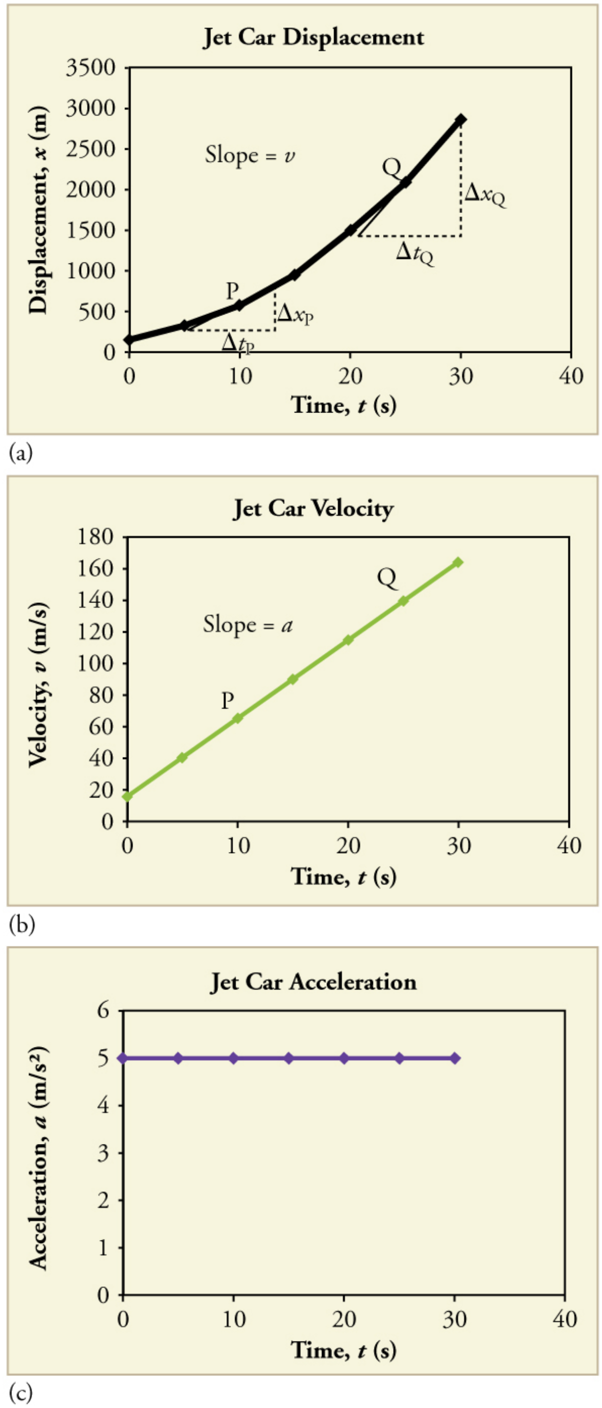


Figure 2.48 Graphs of motion of a jet-powered car during the time span when its acceleration is constant. (a) The slope of an x vs. t graph is velocity. This is shown at two points, and the instantaneous velocities obtained are plotted in the next graph. Instantaneous velocity at any point is the slope of the tangent at that point. (b) The slope of the v vs. t graph is constant for this part of the motion, indicating constant acceleration. (c) Acceleration has the constant value of $5.0\ \text{m/s}^2$ over the time interval plotted.

Figure 2.49 A U.S. Air Force jet car speeds down a track. (credit: Matt Trostle, Flickr)

The graph of displacement versus time in **Figure 2.48**(a) is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a displacement-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line tangent to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in **Figure 2.48**(a). If this is done at every point on the curve and the values are plotted against time, then the graph of velocity versus time shown in **Figure 2.48**(b) is obtained. Furthermore, the slope of the graph of velocity versus time is acceleration, which is shown in **Figure 2.48**(c).

Example 2.18 Determining Instantaneous Velocity from the Slope at a Point: Jet Car

Calculate the velocity of the jet car at a time of 25 s by finding the slope of the x vs. t graph in the graph below.

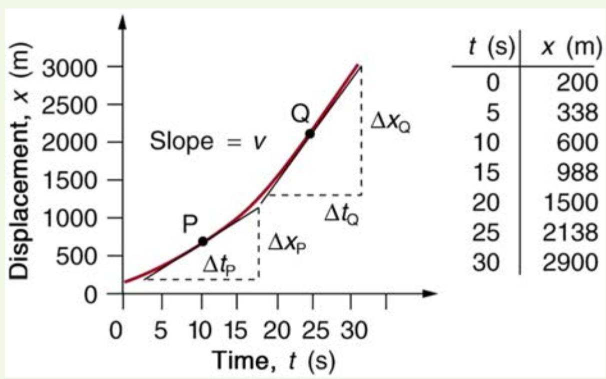


Figure 2.50 The slope of an x vs. t graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point. This principle is illustrated in **Figure 2.50**, where Q is the point at $t = 25\text{ s}$.

Solution

1. Find the tangent line to the curve at $t = 25\text{ s}$.

2. Determine the endpoints of the tangent. These correspond to a position of 1300 m at time 19 s and a position of 3120 m at time 32 s.

3. Plug these endpoints into the equation to solve for the slope, v.

$$\text{slope} = v_\text{Q} = \frac{\Delta x_\text{Q}}{\Delta t_\text{Q}} = \frac{(3120\text{ m} - 1300\text{ m})}{(32\text{ s} - 19\text{ s})} \tag{2.96}$$

Thus,

$$v_\text{Q} = \frac{1820\text{ m}}{13\text{ s}} = 140\text{ m/s}. \tag{2.97}$$

Discussion

This is the value given in this figure's table for v at $t = 25\text{ s}$. The value of 140 m/s for v_Q is plotted in **Figure 2.50**. The entire graph of v vs. t can be obtained in this fashion.

Carrying this one step further, we note that the slope of a velocity versus time graph is acceleration. Slope is rise divided by run; on a v vs. t graph, rise = change in velocity Δv and run = change in time Δt.

The Slope of *v* vs. *t*

The slope of a graph of velocity v vs. time t is acceleration a.

$$\text{slope} = \frac{\Delta v}{\Delta t} = a \tag{2.98}$$

Since the velocity versus time graph in **Figure 2.48**(b) is a straight line, its slope is the same everywhere, implying that acceleration is constant. Acceleration versus time is graphed in **Figure 2.48**(c).

Additional general information can be obtained from **Figure 2.50** and the expression for a straight line, $y = mx + b$.

In this case, the vertical axis y is V, the intercept b is v_0, the slope m is a, and the horizontal axis x is t. Substituting these symbols yields

$$v = v_0 + at. \tag{2.99}$$

A general relationship for velocity, acceleration, and time has again been obtained from a graph. Notice that this equation was also derived algebraically from other motion equations in **Motion Equations for Constant Acceleration in One Dimension**.

It is not accidental that the same equations are obtained by graphical analysis as by algebraic techniques. In fact, an important way to *discover* physical relationships is to measure various physical quantities and then make graphs of one quantity against another to see if they are correlated in any way. Correlations imply physical relationships and might be shown by smooth graphs such as those above. From such graphs, mathematical relationships can sometimes be postulated. Further experiments are then performed to determine the validity of the hypothesized relationships.

Graphs of Motion Where Acceleration is Not Constant

Now consider the motion of the jet car as it goes from 165 m/s to its top velocity of 250 m/s, graphed in **Figure 2.51**. Time again starts at zero, and the initial displacement and velocity are 2900 m and 165 m/s, respectively. (These were the final displacement and velocity of the car in the motion graphed in **Figure 2.48**.) Acceleration gradually decreases from 5.0 m/s^2 to zero when the car hits 250 m/s. The slope of the x vs. t graph increases until $t = 55 \text{ s}$, after which time the slope is constant. Similarly, velocity increases until 55 s and then becomes constant, since acceleration decreases to zero at 55 s and remains zero afterward.

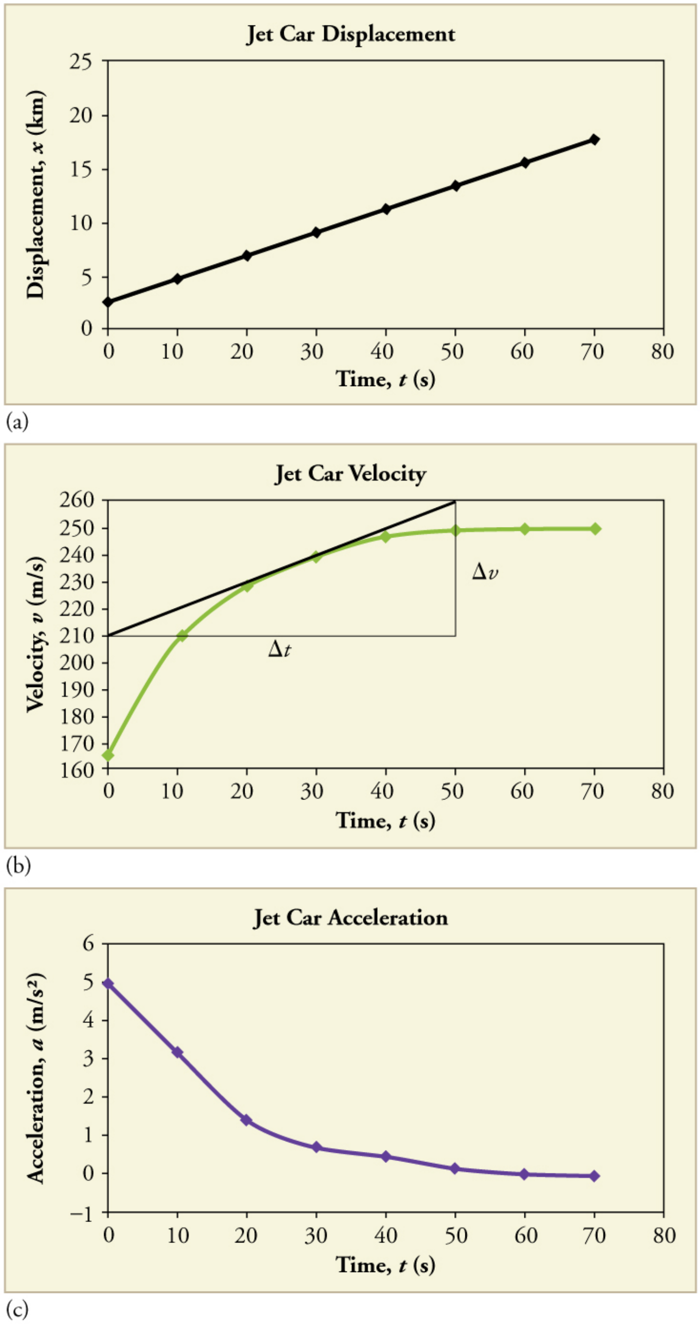


Figure 2.51 Graphs of motion of a jet-powered car as it reaches its top velocity. This motion begins where the motion in **Figure 2.48** ends. (a) The slope of this graph is velocity; it is plotted in the next graph. (b) The velocity gradually approaches its top value. The slope of this graph is acceleration; it is plotted in the final graph. (c) Acceleration gradually declines to zero when velocity becomes constant.

Example 2.19 Calculating Acceleration from a Graph of Velocity versus Time

Calculate the acceleration of the jet car at a time of 25 s by finding the slope of the v vs. t graph in **Figure 2.51**(b).

Strategy

The slope of the curve at $t = 25\text{ s}$ is equal to the slope of the line tangent at that point, as illustrated in **Figure 2.51**(b).

Solution

Determine endpoints of the tangent line from the figure, and then plug them into the equation to solve for slope, a.

$$\text{slope} = \frac{\Delta v}{\Delta t} = \frac{(260\text{ m/s} - 210\text{ m/s})}{(51\text{ s} - 1.0\text{ s})} \tag{2.100}$$

$$a = \frac{50\text{ m/s}}{50\text{ s}} = 1.0\text{ m/s}^2. \tag{2.101}$$

Discussion

Note that this value for a is consistent with the value plotted in **Figure 2.51**(c) at $t = 25\text{ s}$.

A graph of displacement versus time can be used to generate a graph of velocity versus time, and a graph of velocity versus time can be used to generate a graph of acceleration versus time. We do this by finding the slope of the graphs at every point. If the graph is linear (i.e., a line with a constant slope), it is easy to find the slope at any point and you have the slope for every point. Graphical analysis of motion can be used to describe both specific and general characteristics of kinematics. Graphs can also be used for other topics in physics. An important aspect of exploring physical relationships is to graph them and look for underlying relationships.

Check Your Understanding

A graph of velocity vs. time of a ship coming into a harbor is shown below. (a) Describe the motion of the ship based on the graph. (b)What would a graph of the ship's acceleration look like?

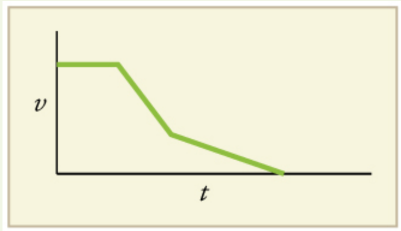


Figure 2.52

Solution

(a) The ship moves at constant velocity and then begins to decelerate at a constant rate. At some point, its deceleration rate decreases. It maintains this lower deceleration rate until it stops moving.

(b) A graph of acceleration vs. time would show zero acceleration in the first leg, large and constant negative acceleration in the second leg, and constant negative acceleration.

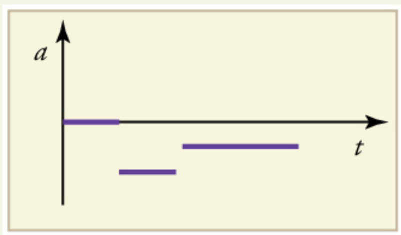


Figure 2.53

Glossary

acceleration due to gravity: acceleration of an object as a result of gravity

acceleration: the rate of change in velocity; the change in velocity over time

average acceleration: the change in velocity divided by the time over which it changes

average speed: distance traveled divided by time during which motion occurs

average velocity: displacement divided by time over which displacement occurs

deceleration: acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

dependent variable: the variable that is being measured; usually plotted along the y-axis

displacement: the change in position of an object

distance traveled: the total length of the path traveled between two positions

distance: the magnitude of displacement between two positions

elapsed time: the difference between the ending time and beginning time

free-fall: the state of movement that results from gravitational force only

independent variable: the variable that the dependent variable is measured with respect to; usually plotted along the x-axis

instantaneous acceleration: acceleration at a specific point in time

instantaneous speed: magnitude of the instantaneous velocity

instantaneous velocity: velocity at a specific instant, or the average velocity over an infinitesimal time interval

kinematics: the study of motion without considering its causes

model: simplified description that contains only those elements necessary to describe the physics of a physical situation

position: the location of an object at a particular time

scalar: a quantity that is described by magnitude, but not direction

slope: the difference in y -value (the rise) divided by the difference in x -value (the run) of two points on a straight line

time: change, or the interval over which change occurs

vector: a quantity that is described by both magnitude and direction

y-intercept: the y- value when $x = 0$, or when the graph crosses the y -axis

Section Summary

2.1 Displacement

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.
- In symbols, displacement Δx is defined to be

$$\Delta x = x_\text{f} - x_0,$$

  where x_0 is the initial position and x_f is the final position. In this text, the Greek letter Δ (delta) always means "change in" whatever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.
- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

2.2 Vectors, Scalars, and Coordinate Systems

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

2.3 Time, Velocity, and Speed

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is

$$\Delta t = t_\text{f} - t_0,$$

  where t_f is the final time and t_0 is the initial time. The initial time is often taken to be zero, as if measured with a stopwatch; the elapsed time is then just t .
- Average velocity $\bar{v}$ is defined as displacement divided by the travel time. In symbols, average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_\text{f} - x_0}{t_\text{f} - t_0}.$$

- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity v is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.
- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

2.4 Acceleration

- Acceleration is the rate at which velocity changes. In symbols, **average acceleration** $\bar{a}$ is

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_\text{f} - v_0}{t_\text{f} - t_0}.$$

- The SI unit for acceleration is m/s^2 .
- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration a is the acceleration at a specific instant in time.
- Deceleration is an acceleration with a direction opposite to that of the velocity.

2.5 Motion Equations for Constant Acceleration in One Dimension

- To simplify calculations we take acceleration to be constant, so that $\bar{a} = a$ at all times.
- We also take initial time to be zero.

- Initial position and velocity are given a subscript 0; final values have no subscript. Thus,

$$\left.\begin{array}{lcl}\Delta t &=& t\\ \Delta x &=& x - x_0\\ \Delta v &=& v - v_0\end{array}\right\}$$

- The following kinematic equations for motion with constant a are useful:

$$x = x_0 + \bar{v}\,t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

- In vertical motion, y is substituted for x.

2.6 Problem-Solving Basics for One-Dimensional Kinematics

- *The six basic problem solving steps for physics are:*
  Step 1. Examine the situation to determine which physical principles are involved.
  Step 2. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).
  Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).
  Step 4. Find an equation or set of equations that can help you solve the problem.
  Step 5. Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.
  Step 6. Check the answer to see if it is reasonable: Does it make sense?

2.7 Falling Objects

- An object in free-fall experiences constant acceleration if air resistance is negligible.
- On Earth, all free-falling objects have an acceleration due to gravity g, which averages

$$g = 9.80\text{ m/s}^2.$$

- Whether the acceleration *a* should be taken as $+g$ or $-g$ is determined by your choice of coordinate system. If you choose the upward direction as positive, $a = -g = -9.80\text{ m/s}^2$ is negative. In the opposite case, $a = +\text{g} = 9.80\text{ m/s}^2$ is positive. Since acceleration is constant, the kinematic equations above can be applied with the appropriate $+g$ or $-g$ substituted for a.
- For objects in free-fall, up is normally taken as positive for displacement, velocity, and acceleration.

2.8 Graphical Analysis of One-Dimensional Motion

- Graphs of motion can be used to analyze motion.
- Graphical solutions yield identical solutions to mathematical methods for deriving motion equations.
- The slope of a graph of displacement x vs. time t is velocity v.
- The slope of a graph of velocity v vs. time t graph is acceleration a.
- Average velocity, instantaneous velocity, and acceleration can all be obtained by analyzing graphs.

Conceptual Questions

2.1 Displacement

1. Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.

2. Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?

3. Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to $50\ \mu\text{m/s}\ \left(50\times10^{-6}\text{ m/s}\right)$ have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

2.2 Vectors, Scalars, and Coordinate Systems

4. A student writes, "*A bird that is diving for prey has a speed of* $-\ 10\ m/s$." What is wrong with the student's statement? What has the student actually described? Explain.

5. What is the speed of the bird in **Exercise 2.4**?

6. Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.

7. A weather forecast states that the temperature is predicted to be -5°C the following day. Is this temperature a vector or a scalar quantity? Explain.

2.3 Time, Velocity, and Speed

8. Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

9. There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.

10. Does a car's odometer measure position or displacement? Does its speedometer measure speed or velocity?

11. If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?

12. How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

2.4 Acceleration

13. Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.

14. Is it possible for velocity to be constant while acceleration is not zero? Explain.

15. Give an example in which velocity is zero yet acceleration is not.

16. If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

17. Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

2.6 Problem-Solving Basics for One-Dimensional Kinematics

18. What information do you need in order to choose which equation or equations to use to solve a problem? Explain.

19. What is the last thing you should do when solving a problem? Explain.

2.7 Falling Objects

20. What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?

21. An object that is thrown straight up falls back to Earth. This is one-dimensional motion. (a) When is its velocity zero? (b) Does its velocity change direction? (c) Does the acceleration due to gravity have the same sign on the way up as on the way down?

22. Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.

23. If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?

24. The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about 1/6 that of the Earth)?

25. How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about 1/6 of g on Earth)?

2.8 Graphical Analysis of One-Dimensional Motion

26. (a) Explain how you can use the graph of position versus time in Figure 2.54 to describe the change in velocity over time. Identify (b) the time (t_a , t_b , t_c , t_d , or t_e) at which the instantaneous velocity is greatest, (c) the time at which it is zero, and (d) the time at which it is negative.

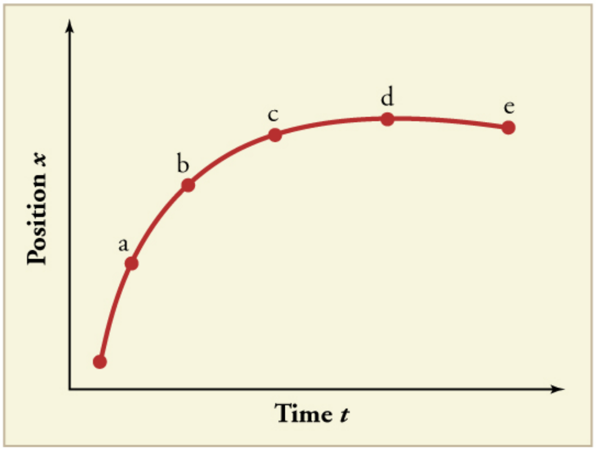


Figure 2.54

27. (a) Sketch a graph of velocity versus time corresponding to the graph of displacement versus time given in Figure 2.55. (b) Identify the time or times (t_a , t_b , t_c , etc.) at which the instantaneous velocity is greatest. (c) At which times is it zero? (d) At which times is it negative?

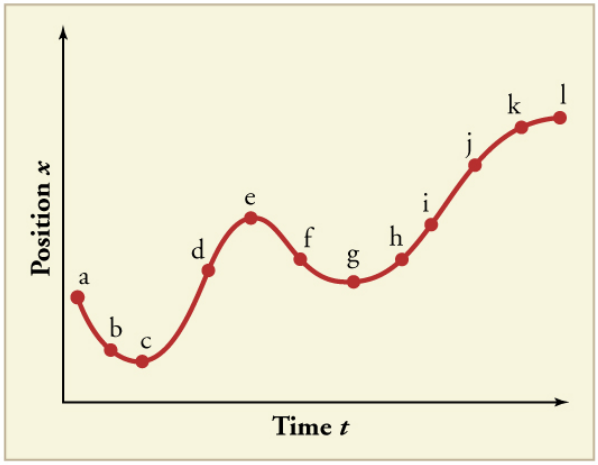


Figure 2.55

28. (a) Explain how you can determine the acceleration over time from a velocity versus time graph such as the one in Figure 2.56. (b) Based on the graph, how does acceleration change over time?

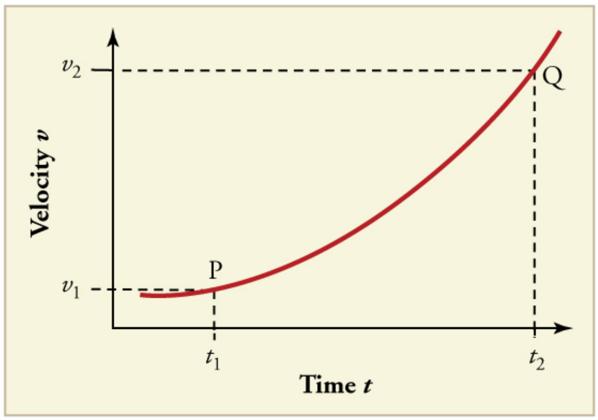


Figure 2.56

29. (a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in Figure 2.57. (b) Identify the time or times (t_a , t_b , t_c , etc.) at which the acceleration is greatest. (c) At which times is it zero? (d) At which times is it negative?

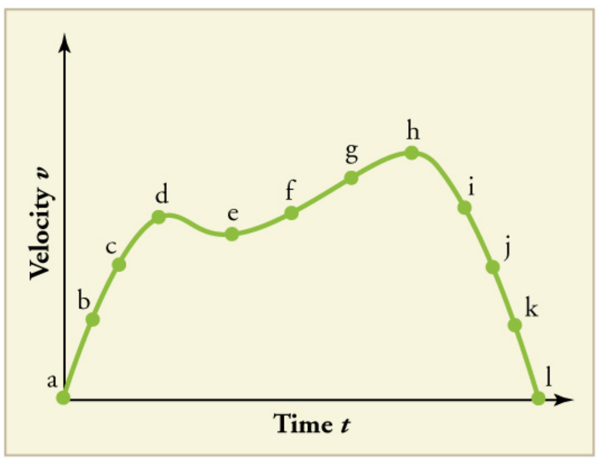


Figure 2.57

30. Consider the velocity vs. time graph of a person in an elevator shown in Figure 2.58. Suppose the elevator is initially at rest. It then accelerates for 3 seconds, maintains that velocity for 15 seconds, then decelerates for 5 seconds until it stops. The acceleration for the entire trip is not constant so we cannot use the equations of motion from Motion Equations for Constant Acceleration in One Dimension for the complete trip. (We could, however, use them in the three individual sections where acceleration is a constant.) Sketch graphs of (a) position vs. time and (b) acceleration vs. time for this trip.

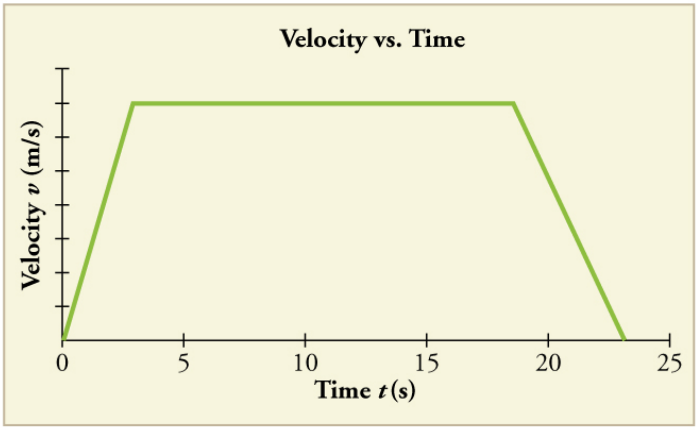


Figure 2.58

31. A cylinder is given a push and then rolls up an inclined plane. If the origin is the starting point, sketch the position, velocity, and acceleration of the cylinder vs. time as it goes up and then down the plane.

Problems & Exercises

2.1 Displacement

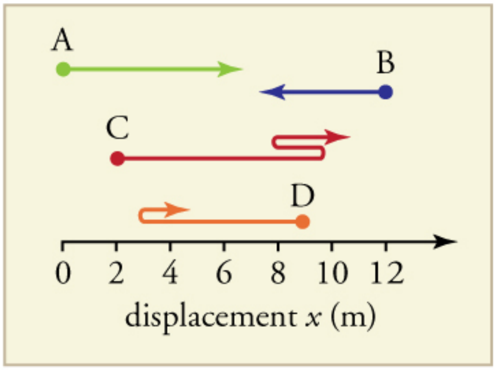


Figure 2.59

1. Find the following for path A in Figure 2.59: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

2. Find the following for path B in Figure 2.59: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

3. Find the following for path C in Figure 2.59: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

4. Find the following for path D in Figure 2.59: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

2.3 Time, Velocity, and Speed

5. (a) Calculate Earth's average speed relative to the Sun. (b) What is its average velocity over a period of one year?

6. A helicopter blade spins at exactly 100 revolutions per minute. Its tip is 5.00 m from the center of rotation. (a) Calculate the average speed of the blade tip in the helicopter's frame of reference. (b) What is its average velocity over one revolution?

7. The North American and European continents are moving apart at a rate of about 3 cm/y. At this rate how long will it take them to drift 500 km farther apart than they are at present?

8. Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?

9. On May 26, 1934, a streamlined, stainless steel diesel train called the Zephyr set the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?

10. Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by $3.84\times10^6\text{ m}$ (1%)?

11. A student drove to the university from her home and noted that the odometer reading of her car increased by 12.0 km. The trip took 18.0 min. (a) What was her average speed? (b) If the straight-line distance from her home to the university is 10.3 km in a direction 25.0° south of east, what was her average velocity? (c) If she returned home by the same path 7 h 30 min after she left, what were her average speed and velocity for the entire trip?

12. The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance?

13. Conversations with astronauts on the lunar surface were characterized by a kind of echo in which the earthbound person's voice was so loud in the astronaut's space helmet that it was picked up by the astronaut's microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light $(3.00\times10^8\text{ m/s})$.

14. A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity (a) for each of the three intervals and (b) for the entire motion.

15. The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom, as having a single electron in a circular orbit $1.06\times10^{-10}\text{ m}$ in diameter. (a) If the average speed of the electron in this orbit is known to be $2.20\times10^6\text{ m/s}$, calculate the number of revolutions per second it makes about the nucleus. (b) What is the electron's average velocity?

2.4 Acceleration

16. A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

17. Professional Application

Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration. Express each in multiples of g (9.80 m/s^2) by taking its ratio to the acceleration of gravity.

18. A commuter backs her car out of her garage with an acceleration of 1.40 m/s^2. (a) How long does it take her to reach a speed of 2.00 m/s? (b) If she then brakes to a stop in 0.800 s, what is her deceleration?

19. Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in m/s^2 and in multiples of g (9.80 m/s^2)?

2.5 Motion Equations for Constant Acceleration in One Dimension

20. An Olympic-class sprinter starts a race with an acceleration of 4.50 m/s^2. (a) What is her speed 2.40 s later? (b) Sketch a graph of her position vs. time for this period.

21. A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is $2.10\times10^4\text{ m/s}^2$, and 1.85 ms $(1\text{ ms} = 10^{-3}\text{ s})$ elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?

22. A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of $6.20\times10^5\text{ m/s}^2$ for $8.10\times10^{-4}\text{ s}$. What is its muzzle velocity (that is, its final velocity)?

23. (a) A light-rail commuter train accelerates at a rate of 1.35 m/s^2. How long does it take to reach its top speed of 80.0 km/h, starting from rest? (b) The same train ordinarily decelerates at a rate of 1.65 m/s^2. How long does it take to come to a stop from its top speed? (c) In emergencies the train can decelerate more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency deceleration in m/s^2?

24. While entering a freeway, a car accelerates from rest at a rate of 2.40 m/s^2 for 12.0 s. (a) Draw a sketch of the situation. (b) List the knowns in this problem. (c) How far does the car travel in those 12.0 s? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable. (d) What is the car's final velocity? Solve for this unknown in the same manner as in part (c), showing all steps explicitly.

25. At the end of a race, a runner decelerates from a velocity of 9.00 m/s at a rate of 2.00 m/s^2. (a) How far does she travel in the next 5.00 s? (b) What is her final velocity? (c) Evaluate the result. Does it make sense?

26. Professional Application:

Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?

27. In a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes $3.33\times10^{-2} \text{ s}$, calculate the distance over which the puck accelerates.

28. A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s. (a) What is its average acceleration? (b) How far does it travel in that time?

29. Freight trains can produce only relatively small accelerations and decelerations. (a) What is the final velocity of a freight train that accelerates at a rate of 0.0500 m/s^2 for 8.00 min, starting with an initial velocity of 4.00 m/s? (b) If the train can slow down at a rate of 0.550 m/s^2, how long will it take to come to a stop from this velocity? (c) How far will it travel in each case?

30. A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m. (a) How long did the acceleration last? (b) Calculate the acceleration.

31. A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of 0.350 m/s^2, how far will it travel before becoming airborne? (b) How long does this take?

32. Professional Application:

A woodpecker's brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in m/s^2 and in multiples of $g \left(g = 9.80 \text{ m/s}^2\right)$. (b) Calculate the stopping time. (c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain's deceleration, expressed in multiples of g?

33. An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m. (a) What is his deceleration? (b) How long does the collision last?

34. In World War II, there were several reported cases of airmen who jumped from their flaming airplanes with no parachute to escape certain death. Some fell about 20,000 feet (6000 m), and some of them survived, with few life-threatening injuries. For these lucky pilots, the tree branches and snow drifts on the ground allowed their deceleration to be relatively small. If we assume that a pilot's speed upon impact was 123 mph (54 m/s), then what was his deceleration? Assume that the trees and snow stopped him over a distance of 3.0 m.

35. Consider a grey squirrel falling out of a tree to the ground. (a) If we ignore air resistance in this case (only for the sake of this problem), determine a squirrel's velocity just before hitting the ground, assuming it fell from a height of 3.0 m. (b) If the squirrel stops in a distance of 2.0 cm through bending its limbs, compare its deceleration with that of the airman in the previous problem.

36. An express train passes through a station. It enters with an initial velocity of 22.0 m/s and decelerates at a rate of 0.150 m/s^2 as it goes through. The station is 210 m long. (a) How long is the nose of the train in the station? (b) How fast is it going when the nose leaves the station? (c) If the train is 130 m long, when does the end of the train leave the station? (d) What is the velocity of the end of the train as it leaves?

37. Dragsters can actually reach a top speed of 145 m/s in only 4.45 s—considerably less time than given in **Example 2.10** and **Example 2.11**. (a) Calculate the average acceleration for such a dragster. (b) Find the final velocity of this dragster starting from rest and accelerating at the rate found in (a) for 402 m (a quarter mile) without using any information on time. (c) Why is the final velocity greater than that used to find the average acceleration? *Hint*: Consider whether the assumption of constant acceleration is valid for a dragster. If not, discuss whether the acceleration would be greater at the beginning or end of the run and what effect that would have on the final velocity.

38. A bicycle racer sprints at the end of a race to clinch a victory. The racer has an initial velocity of 11.5 m/s and accelerates at the rate of 0.500 m/s^2 for 7.00 s. (a) What is his final velocity? (b) The racer continues at this velocity to the finish line. If he was 300 m from the finish line when he started to accelerate, how much time did he save? (c) One other racer was 5.00 m ahead when the winner started to accelerate, but he was unable to accelerate, and traveled at 11.8 m/s until the finish line. How far ahead of him (in meters and in seconds) did the winner finish?

39. In 1967, New Zealander Burt Munro set the world record for an Indian motorcycle, on the Bonneville Salt Flats in Utah, of 183.58 mi/h. The one-way course was 5.00 mi long. Acceleration rates are often described by the time it takes to reach 60.0 mi/h from rest. If this time was 4.00 s, and Burt accelerated at this rate until he reached his maximum speed, how long did it take Burt to complete the course?

40. (a) A world record was set for the men's 100-m dash in the 2008 Olympic Games in Beijing by Usain Bolt of Jamaica. Bolt "coasted" across the finish line with a time of 9.69 s. If we assume that Bolt accelerated for 3.00 s to reach his maximum speed, and maintained that speed for the rest of the race, calculate his maximum speed and his acceleration. (b) During the same Olympics, Bolt also set the world record in the 200-m dash with a time of 19.30 s. Using the same assumptions as for the 100-m dash, what was his maximum speed for this race?

2.7 Falling Objects

Assume air resistance is negligible unless otherwise stated.

41. Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, and (d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be $y_0 = 0$.

42. Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, (d) 2.00, and (e) 2.50 s for a rock thrown straight down with an initial velocity of 14.0 m/s from the Verrazano Narrows Bridge in New York City. The roadway of this bridge is 70.0 m above the water.

43. A basketball referee tosses the ball straight up for the starting tip-off. At what velocity must a basketball player leave the ground to rise 1.25 m above the floor in an attempt to get the ball?

44. A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of 1.40 m/s and observes that it takes 1.8 s to reach the water. (a) List the knowns in this problem. (b) How high above the water was the preserver released? Note that the downdraft of the helicopter reduces the effects of air resistance on the falling life preserver, so that an acceleration equal to that of gravity is reasonable.

45. A dolphin in an aquatic show jumps straight up out of the water at a velocity of 13.0 m/s. (a) List the knowns in this problem. (b) How high does his body rise above the water? To solve this part, first note that the final velocity is now a known and identify its value. Then identify the unknown, and discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking units, and discuss whether the answer is reasonable. (c) How long is the dolphin in the air? Neglect any effects due to his size or orientation.

46. A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of 4.00 m/s, and her takeoff point is 1.80 m above the pool. (a) How long are her feet in the air? (b) What is her highest point above the board? (c) What is her velocity when her feet hit the water?

47. (a) Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s. (b) How long would it take to reach the ground if it is thrown straight down with the same speed?

48. A very strong, but inept, shot putter puts the shot straight up vertically with an initial velocity of 11.0 m/s. How long does he have to get out of the way if the shot was released at a height of 2.20 m, and he is 1.80 m tall?

49. You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?

50. A kangaroo can jump over an object 2.50 m high. (a) Calculate its vertical speed when it leaves the ground. (b) How long is it in the air?

51. Standing at the base of one of the cliffs of Mt. Arapiles in Victoria, Australia, a hiker hears a rock break loose from a height of 105 m. He can't see the rock right away but then does, 1.50 s later. (a) How far above the hiker is the rock when he can see it? (b) How much time does he have to move before the rock hits his head?

52. An object is dropped from a height of 75.0 m above ground level. (a) Determine the distance traveled during the first second. (b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground.

53. There is a 250-m-high cliff at Half Dome in Yosemite National Park in California. Suppose a boulder breaks loose from the top of this cliff. (a) How fast will it be going when it strikes the ground? (b) Assuming a reaction time of 0.300 s, how long will a tourist at the bottom have to get out of the way after hearing the sound of the rock breaking loose (neglecting the height of the tourist, which would become negligible anyway if hit)? The speed of sound is 335 m/s on this day.

54. A ball is thrown straight up. It passes a 2.00-m-high window 7.50 m off the ground on its path up and takes 1.30 s to go past the window. What was the ball's initial velocity?

55. Suppose you drop a rock into a dark well and, using precision equipment, you measure the time for the sound of a splash to return. (a) Neglecting the time required for sound to travel up the well, calculate the distance to the water if the sound returns in 2.0000 s. (b) Now calculate the distance taking into account the time for sound to travel up the well. The speed of sound is 332.00 m/s in this well.

56. A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms $(8.00\times10^{-5}\text{ s})$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

57. A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground.

58. A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms $(3.50\times10^{-3}\text{ s})$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

2.8 Graphical Analysis of One-Dimensional Motion

Note: There is always uncertainty in numbers taken from graphs. If your answers differ from expected values, examine them to see if they are within data extraction uncertainties estimated by you.

59. (a) By taking the slope of the curve in **Figure 2.60**, verify that the velocity of the jet car is 115 m/s at $t = 20\text{ s}$. (b) By taking the slope of the curve at any point in **Figure 2.61**, verify that the jet car's acceleration is 5.0 m/s^2.

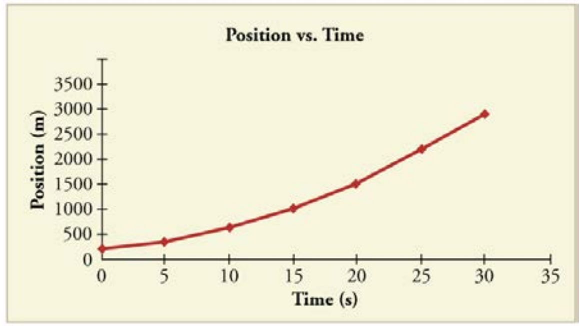


Figure 2.60

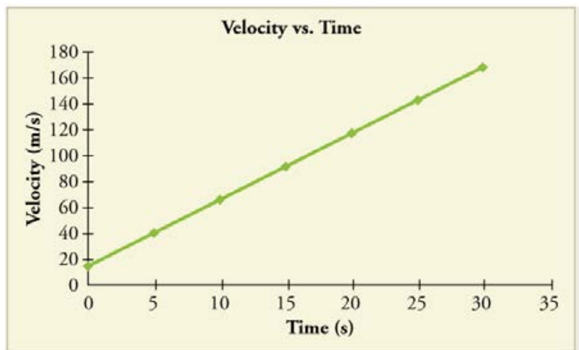


Figure 2.61

60. Take the slope of the curve in **Figure 2.62** to verify that the velocity at $t = 10\text{ s}$ is 207 m/s.

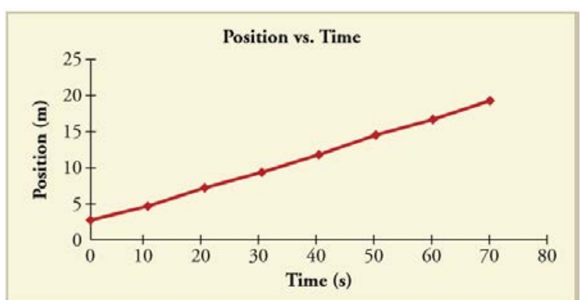


Figure 2.62

61. Take the slope of the curve in **Figure 2.62** to verify that the velocity at $t = 30.0\text{ s}$ is 238 m/s.

62. By taking the slope of the curve in **Figure 2.63**, verify that the acceleration is 3.2 m/s^2 at $t = 10\text{ s}$.

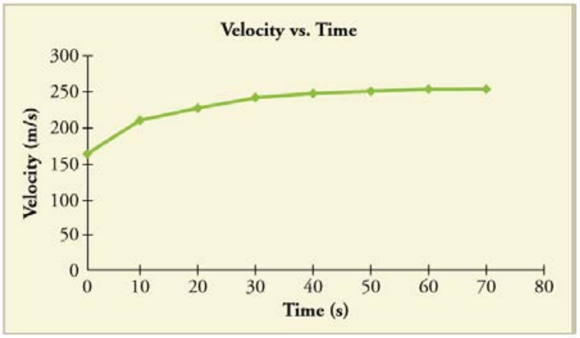


Figure 2.63

63. Construct the displacement graph for the subway shuttle train as shown in Figure 2.48(a). You will need to use the information on acceleration and velocity given in the examples for this figure.

64. (a) Take the slope of the curve in Figure 2.64 to find the jogger's velocity at $t = 2.5\text{ s}$. (b) Repeat at 7.5 s. These values must be consistent with the graph in Figure 2.65.

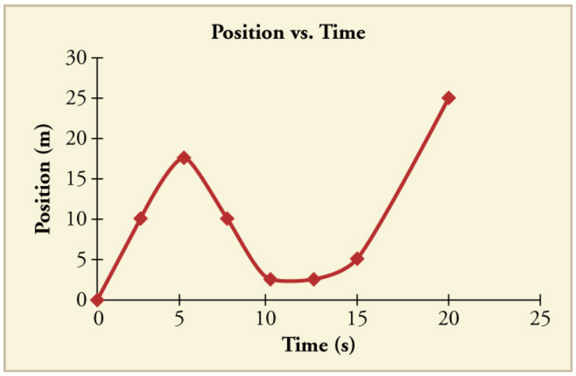


Figure 2.64

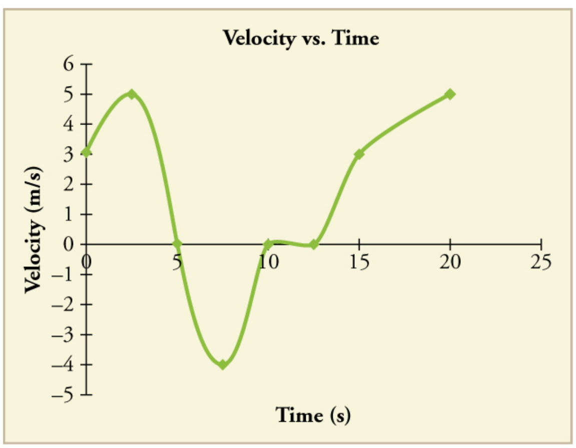


Figure 2.65

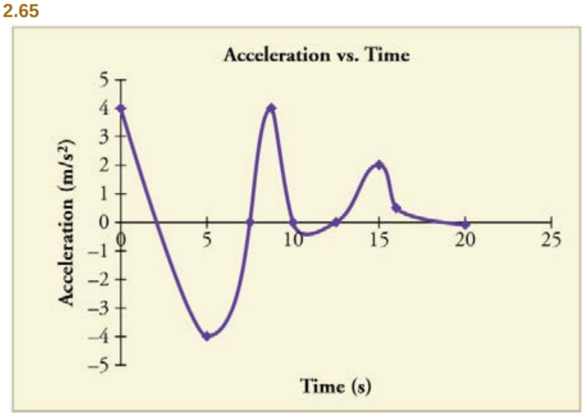


Figure 2.66

65. A graph of $v(t)$ is shown for a world-class track sprinter in a 100-m race. (See Figure 2.67). (a) What is his average velocity for the first 4 s? (b) What is his instantaneous velocity at $t = 5\text{ s}$? (c) What is his average acceleration between 0 and 4 s? (d) What is his time for the race?

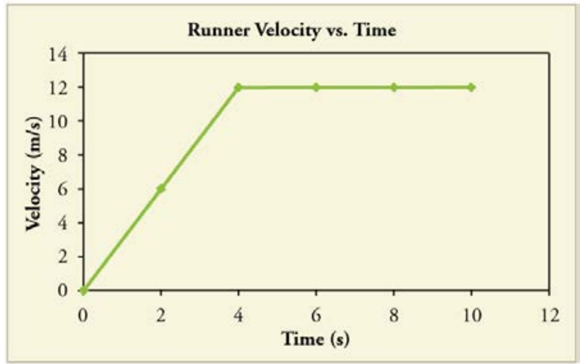


Figure 2.67

66. Figure 2.68 shows the displacement graph for a particle for 5 s. Draw the corresponding velocity and acceleration graphs.

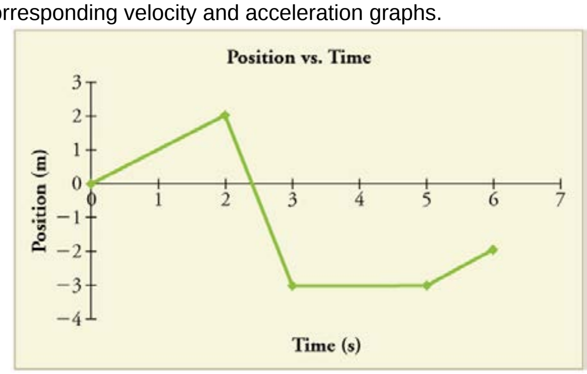


Figure 2.68