# Experiment 7: Circular Motion and Centripetal Force

## I. About the Experiment

In the experiment we will study the motion of an object moving around a circular path with a constant tangential speed. Remember that acceleration is the rate of change of the velocity **vector** and thus even though the tangential speed of an object may be constant, it is still accelerating since the direction of the velocity **vector** is continuously changing. The acceleration of an object moving around a circular path at a constant tangential speed is called centripetal (center seeking) acceleration and the direction of this acceleration is always towards the center of the circle the object is moving around. The magnitude of the centripetal acceleration is given by

$$a_c = \left|\vec{\mathbf{a}}_c\right| = \frac{\left|\vec{\mathbf{v}}_t\right|^2}{R} = \frac{v_t^2}{R} \qquad \text{Eqn. 1}$$

where R is the radius of the circular path.

Now Newton's second law is

$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = m\frac{d\vec{\mathbf{v}}}{dt} \qquad \text{Eqn. 2}$$

From Equations 1 and 2 we see that there **must** be a net force on an object moving around a circular path at a constant speed. This net force is called the centripetal force. The centripetal force always points towards the center of the circle the object is moving around. By combining Equations 1 and 2 we obtain the magnitude of the centripetal force.

$$F_c = \left|\vec{\mathbf{F}}_c\right| = ma_c = m\frac{v_t^2}{R} \qquad \text{Eqn. 3}$$

Figure 1 illustrates the relationship of these vectors. Note: regardless of the instantaneous direction of $\vec{\mathbf{v}}_t$, $\vec{\mathbf{F}}_c$ always points towards the center of the circle.

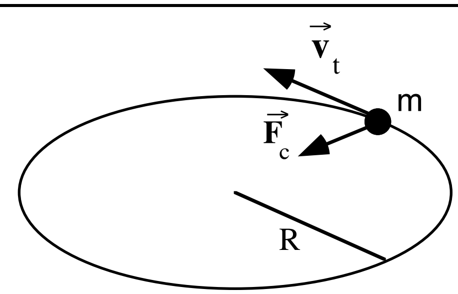


Figure 1

**The goal of this experiment is to test Equation 3 by <u>separately</u> determining the centripetal force and the centripetal acceleration.**

## II. Experimental Set-up

In this experiment a mass (m) is rotated about an axis of rotation at a radius, *R*. Throughout this experiment, the centripetal force and the mass of the object are held constant. The radius of rotation is varied for each trial. A constant centripetal force is maintained by adjusting the tangential speed of the rotating apparatus.

Refer to Figure 2. In this apparatus the tension in the spring supplies the centripetal force while the hooked mass (m) is rotating. This force is monitored during rotation by watching the indicating disc. By rotating the apparatus at a speed that keeps the indicating disc aligned with the reference bracket, both the centripetal force and the radius of rotation are held at constant (preset) values. The centripetal force and the radius of rotation are preset in the following manner.

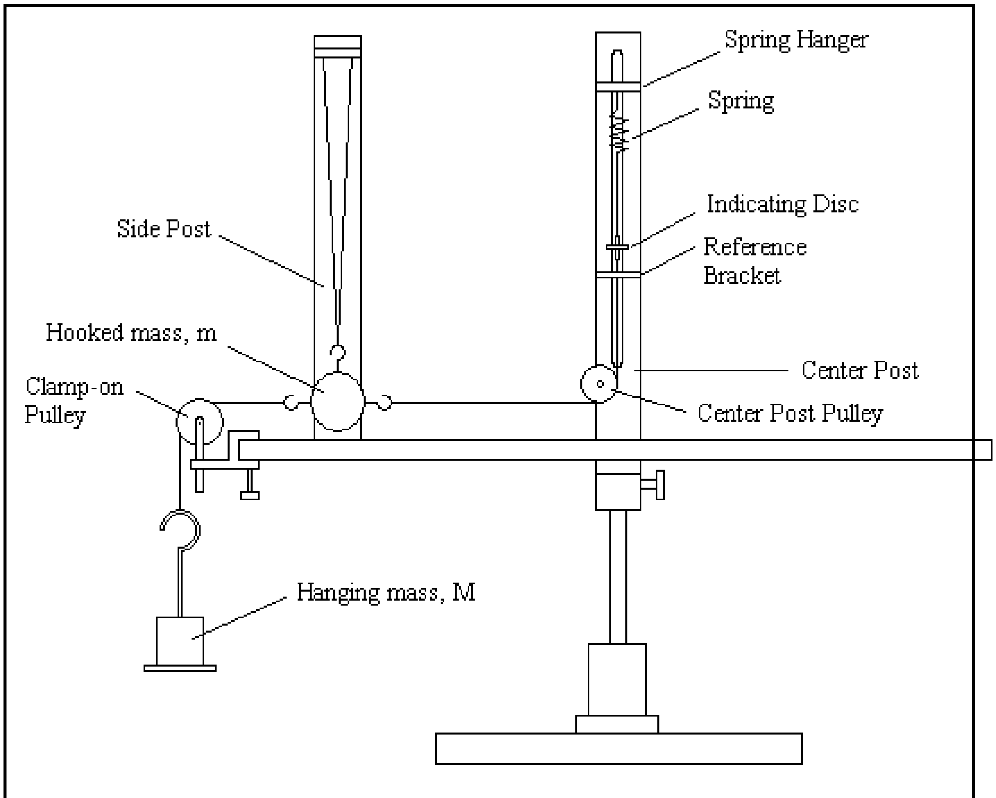


Figure 2

While the apparatus is held stationary, a hanging mass (M) is attached to the hooked mass (m) by means of a clamp-on pulley. This applies a static force $F_s = Mg$ to the spring. The radius of rotation ($R$) is set by changing the position of the side post relative to the center post. The spring hanger is adjusted so that the hooked mass (m) hangs vertically. Finally, the reference bracket is adjusted to coincide with the indicating disc. The hanging mass (M) and clamp-on pulley are removed from the apparatus. This completes the stationary adjustment process.

Now, as the apparatus is rotated, the spring supplies the centripetal force required to keep the hooked mass (m) moving in a uniform circle. Increasing the tangential speed increases the needed centripetal force (refer to equation 3). When the tangential speed is fast enough to align the indicating disc with the reference bracket, the centripetal force will be the same as the static force $F_s = Mg$. This speed is maintained while a number of revolutions are counted and timed. **Note that the apparatus is never rotated with the hanging mass (M) or the clamp-on pulley attached**. The average tangential speed $v_t$ of the hanging mass is determined by timing how long it takes the mass to rotate through a given number of revolutions. Multiplying the number of revolutions by $2\pi R$ gives the total distance traveled by the mass. If the total number of revolutions is given by $N$, and the total amount of time required to make $N$ revolutions is given by $T$, then

$$v_t = \frac{2\pi RN}{T} \qquad \text{Eqn. 4}$$

Combining equations 3 and 4 yields

$$F_c = m(2\pi N)^2 \frac{R}{T^2}$$

which can be written as

$$T^2 = \frac{m(2\pi N)^2}{F_C} R \qquad \text{Eqn. 5}$$

Thus the centripetal force can be calculated by determining the slope of the $T^2$ vs. $R$ curve.

## III. Procedures

1. Level the apparatus in the following manner. Place a weight of about 1 kg on the end of the rotating platform (place it on the same end that the hooked mass (m) hangs from). The platform is now intentionally out of balance. Adjust two of the leveling screws such that the weighted end of the platform lines up over the third leveling screw. Now, gently turn the platform 90 degrees and adjust the third screw so that the platform remains in this new position. When properly balanced, the platform will remain at rest in any position. Remove the 1 kg weight.

2. Check the position of the center post to be sure it is aligned with the axis of rotation. If it is out of alignment, adjust its position so that the reference marks (on the back side of the center post) line up.

3. Weigh the hooked mass (m) assembly and record this on the data sheet.

4. Hang the hooked mass (m) on the apparatus and connect it to the string from the center post spring. Be sure the connecting string rides around the center post pulley.

5. Attach the clamp-on pulley to the platform on the end closest to the side post. Attach a string to the hooked mass (m) and pass this over the clamp-on pulley. Hang a 50 gram mass hanger on this string. Now add a known amount of mass (M) to the hanger. This is the hanging mass that will be used for all of the trials. Do not exceed 150 grams. This establishes a constant force on the spring. Record the total amount of mass (don't forget the mass of the hanger) on the data sheet.

6. Select a radius ($R$ ) by positioning the side post while measuring the distance between the white reference lines on the side and center posts. Make sure the side post is vertical and tighten its positioning screw. Do not change the position of the center post. Record this radius on the data sheet.

7. The hooked mass (m) must hang vertically. This is achieved by adjusting the vertical position of the spring hanger on the center post until the hooked mass is aligned with the white line on the side post. If the spring hanger doesn't have enough range of adjustment go back to step 6 and select a new radius. When the hooked mass (m) is vertical, adjust the indicator bracket on the center post to align it with the white indicating disk. NOTE: While performing the adjustment operations in steps 6 and 7, do not change the position of the center post. It must remain aligned with the axis of rotation as described in step 2 above.

8. Remove the hanging mass (M) and the clamp-on pulley. Gently rotate the apparatus and increase speed until the indicating disc lines up with the reference bracket. This assures us that hooked mass is vertical, hence the radius of rotation is the same as measured in step 6. Select a number ($N$) of rotations to time. A good value for $N$ is 10. Record this on the data sheet. It is important to use the same value of $N$ for each run in each trial. Time how long it takes to complete $N$ rotations, making sure that the indicating disc is lined up with the reference bracket at all times

during the measurement, and record this on the data sheet. Repeat the timing procedure for two more runs so that an average $T$ can be determined.

**Important notes**:
1. Never rotate the apparatus with the hanging mass (M) or clamp-on pulley attached.
2. Use the same hanging mass (M) for each trial.
3. Use the same number of rotations $N$ for each measurement.
4. Repeat steps 5 through 8 four more times using a different radius of rotation ($R$ ) for each trial.

Use this data sheet to record your data.

Mass of hooked mass (m) __________
Mass of hanging mass (M) __________
Number of rotations (N) __________

| Trial | Radius of path (R) | time (T) to make N rotations run 1 | run 2 | run 3 |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |

## IV. Calculations and Analysis

1. Enter all of your data into an Excel spreadsheet and use the spreadsheet to calculate for each trial:
   a. The average (for the 3 runs) of the time $T$ required to make N rotations.
   b. The average (for the 3 runs) of the time squared $T^2$.
2. Calculate the value for the static force $F_s = Mg$. **Note: this a direct determination of the centripetal force.**
3. Plot a graph of $T^2$ vs. $R$ (figure 4).
4. Calculate the slope of the graph of $T^2$ vs. $R$ using one of the following methods:

   a. Best fit straight line using LINEST(y_values,x_values): The easiest way to find the best fit straight line is to use Excel's built in function LINEST(y_values,x_values). This function returns the slope and intercept as a two element array. These elements are selected by the INDEX function. Refer to figure 5 for an example of how to use the LINEST and INDEX functions to obtain the slope and intercepts from a set of data. Note that in this experiment, the x and y-intercepts have no physical meaning, only the slope is used. The LINEST function uses the method of least squares to determine the best fit straight line for a set of data.

   b. Direct use of a least squares algorithm: Some spreadsheets do not have a LINEST or equivalent function. In this case a direct implementation of the method of least squares must be used. Refer to section 12-VI for more information about this.

7. Calculate the value for the centripetal force $F_c$ using the slope of the graph of $T^2$ vs. $R$ and equation 5. The slope $= \dfrac{m(2\pi N)^2}{F_c}$. **Note: this a determination of the centripetal force by means of measuring the centripetal acceleration.**

8. Calculate the unbiased percent difference between the results of $F_C$ and $F_S$

Figure 3 shows a suggested form for your spreadsheet. Cell C1 contains the mass of the hooked mass (m), cell C2 the mass of the hanging mass (M), and cell C3 the number of rotations (*N*). When refering to these values you probably want to use absolute reference (see appendix A3.6). Cells F7:F11 should have formulas to calculate the average time of runs 1-3. Likewise, cells G7:G11 should have formulas to calculate the square of the average time. Cell G12 should use the formula for $F_s$ given in 2 above (be sure to use the value for the hanging mass (M) here). Cell G14 should use equation 5 along with the appropriate values (remember to take advantage of absolute references) for m, N, and the slope of the graph to calculate the centripetal force.

Figures 4 and 5 show how to use, in general, the INDEX and LINEST functions to calculate the slope as discussed above.

**Caution:** Be sure to use a consistent system of units and/or use appropriate unit conversion factors throughout all of these calculations.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | hooked mass, m | | | | | | |
| 2 | hanging mass, M | | | | | | |
| 3 | # of rotations, N | | | | | | |
| 4 | | | | | | | |
| 5 | | | time T to make N rotations | | | | |
| 6 | trial | radius, R | run 1 | run 2 | run 3 | ave. T | ave. T*T |
| 7 | 1 | | | | | | |
| 8 | 2 | | | | | | |
| 9 | 3 | | | | | | |
| 10 | 4 | | | | | | |
| 11 | 5 | | | | | | |
| 12 | | | | | Spring Force | | |
| 13 | | | | | Slope of R vs T*T | | |
| 14 | | | | | Centripetal Force | | |
| 15 | | | | Unbiased percent difference | | | |

Figure 3

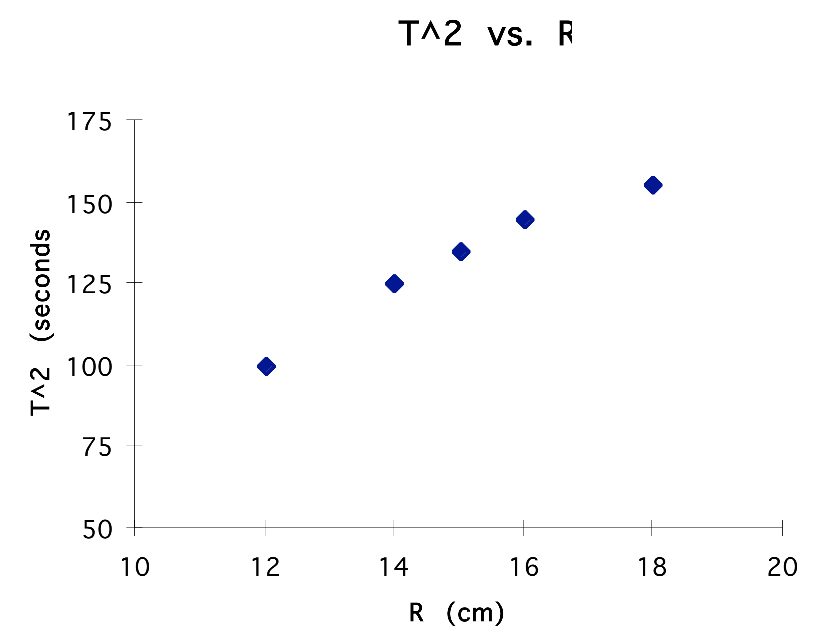


Figure 4

| | A | B |
|---|---|---|
| 1 | x_Values | y_values |
| 2 | 12 | 100 |
| 3 | 14 | 125 |
| 4 | 15 | 135 |
| 5 | 16 | 145 |
| 6 | 18 | 155 |
| 7 | | |
| 8 | slope= | =INDEX(LINEST(B2:B6,A2:A6),1) |
| 9 | y_intercept = | =INDEX(LINEST(B2:B6,A2:A6),2) |
| 10 | x_intercept | =-B9/B8 |

Figure 5

## V. Questions

1. If you were asked "what force keeps a satellite in its orbit around the earth", what would you say? What would happen to the satellite if that force were suddenly removed?
2. Where should you stand on the earth's surface to experience the most centripetal acceleration? The least? Hint 1: how long does it take for the earth to make 1 revolution about its axis of rotation? Hint 2: the R in equation 4 is the distance from the **axis of rotation** to the mass.
3. If you should buy a quantity of gold in Mexico and weigh it carefully on a spring scale, would the same quantity of gold weigh more, less, or the same if weighed on the same spring scale in Alaska? Explain. Note: the weight determined by a spring scale is proportional to the extension of the spring.
4. A bug sits on the very top of a freshly waxed bowling ball. It loses its footing and slides freely down the side of the ball. Explain why the bug will leave the surface <u>before</u> it falls halfway down the ball.
5. When constructing a roller coaster, the designer wishes the riders to experience weightlessness as they round the top of the hill. How fast must the car be going if the radius of curvature at the hill top is 20m?