Experiment 10: Static Equilibrium of a Rigid Body

I. About the Experiment

If an object is stationary and remains stationary, it is said to be in static equilibrium. A necessary condition for static equilibrium of an object is that the sum of the forces acting on the object is zero.

$$\vec{\mathbf{F}}_{\text{net}} = \sum_i \vec{\mathbf{F}}_i = 0 \qquad \text{Eqn.1}$$

Note that this means the sum of the components of the forces must individually equal zero.

$$\sum_i \left(\mathbf{F}_x\right)_i = 0 \qquad \sum_i \left(\mathbf{F}_y\right)_i = 0 \qquad \sum_i \left(\mathbf{F}_z\right)_i = 0 \qquad \text{Eqn.2}$$

However, this alone is not sufficient to establish equilibrium in a rigid body which has forces acting on it which are not concurrent, i.e., **not all acting through the same point**. An example of this is a bar which is free to rotate and which has two operating rocket engines mounted on the ends as shown in Figure 1. Even if F_1 and F_2 should have a vector sum of zero the bar would still have an angular acceleration and thus the bar would not be in static equilibrium.

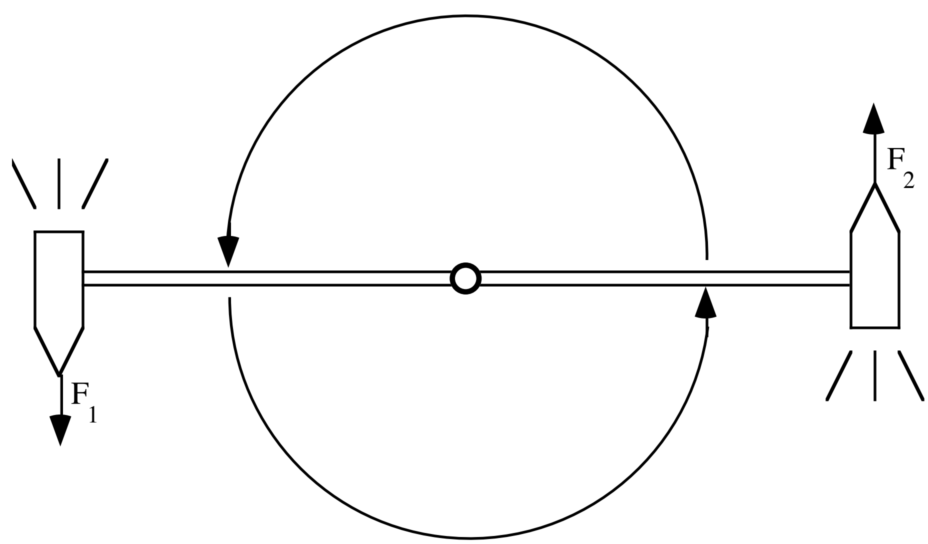

Figure 1

When forces acting on an object do not all act at the same point, the forces are said to produce a **torque** on the object**.** The condition for a rigid body not to be rotating about some point is that the vector sum of the torques, $\vec{\tau}$, about that point be equal to zero, i.e.,

$$\vec{\tau}_{\text{net}} = \sum_i \vec{\tau}_i = 0 \qquad \text{Eqn.3}$$

If both equations 1 and 3 are satisfied for some rigid body, and that body is initially neither translating or rotating, it will remain motionless and is said to be in **static** equilibrium.

You probably have not yet covered the topic of torque in your lecture class and therefore we define torque below. Referring to Figure 2 the quantitative definition of torque is given by

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} \qquad |\vec{\tau}| = |\vec{\mathbf{r}}||\vec{\mathbf{F}}|\sin\theta \qquad \text{Eqn.4}$$

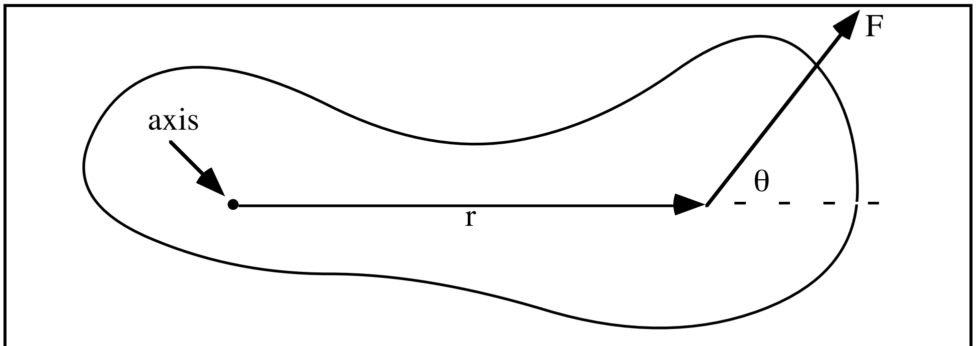

Figure 2

The vector $\vec{\mathbf{r}}$ in equation 4 is the vector **from the POINT about which the torque is being calculated to the point at which the force is acting.** Remember that the order of vectors in a cross product is important! Also notice that $|\mathbf{F}|\sin\theta$ is simply the component of $\vec{\mathbf{F}}$ that is perpendicular to $\vec{\mathbf{r}}$.

It is obvious from Equation 4 that, in general, the magnitude and direction of the torque due to a particular force depends not only the magnitude and direction of that force, but also depends on the point about which the torque is being calculated. However, **if an object is in static equilibrium, the NET TORQUE must be zero about ANY point.**

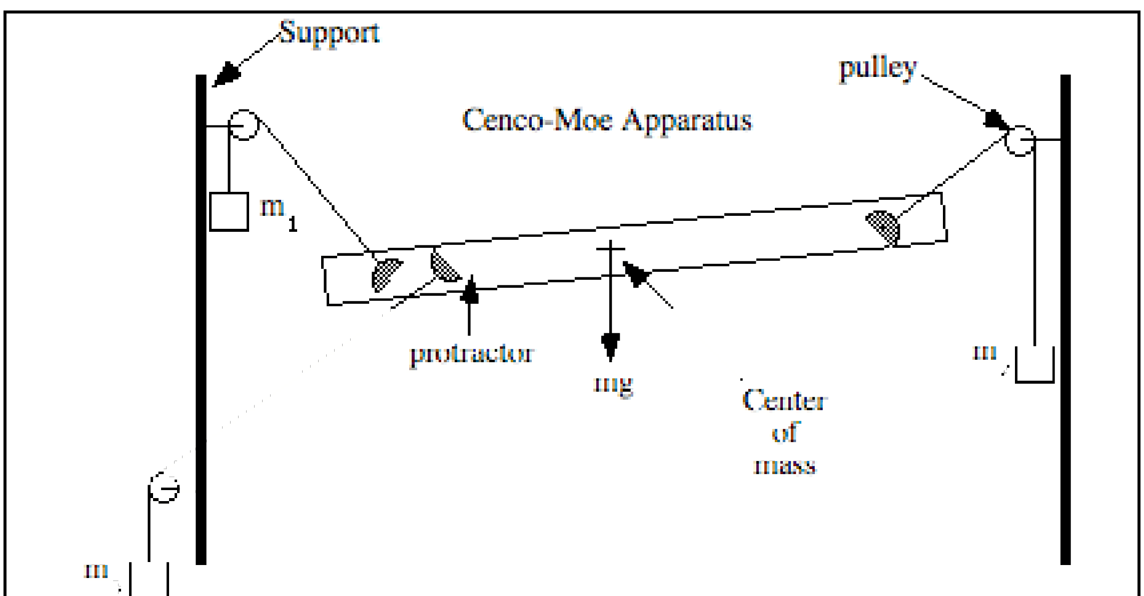

Figure 3

Although torque is a vector quantity, if the rotational motion takes place in a plane, a simple sign convention will suffice in place of vector notation. Torques which would produce a counterclockwise rotation are defined as positive and torques which would produce a clockwise rotation are defined as negative. This greatly simplifies the analysis in this experiment. See specifically step 8 in the Calculations and Analysis section.

II. The Experimental Set-up

In this experiment we will set up an equilibrium condition for a rigid body using a Cenco-Moe apparatus and show experimentally that both the sum of the forces and the sum of the torques are equal to zero to within our experimental uncertainties. The Cenco-Moe apparatus consists of a metal bar on which are mounted four protractors. The apparatus is supported in a vertical plane by means of strings over pulleys on support rods. The protractors permit the measuring of the angles between the bar and the supporting strings. The center of gravity is **NOT** at the center of the bar.

III. Procedures

1. Set up a condition of stable equilibrium on the Cenco-Moe apparatus as shown in Figures 3 and 4. Both the magnitude and direction of the forces may be varied to achieve equilibrium. Be sure that the protractors or the associated pivot arms **are not resting on the horizontal bar**. Jiggle the apparatus to make sure that nothing is stuck.

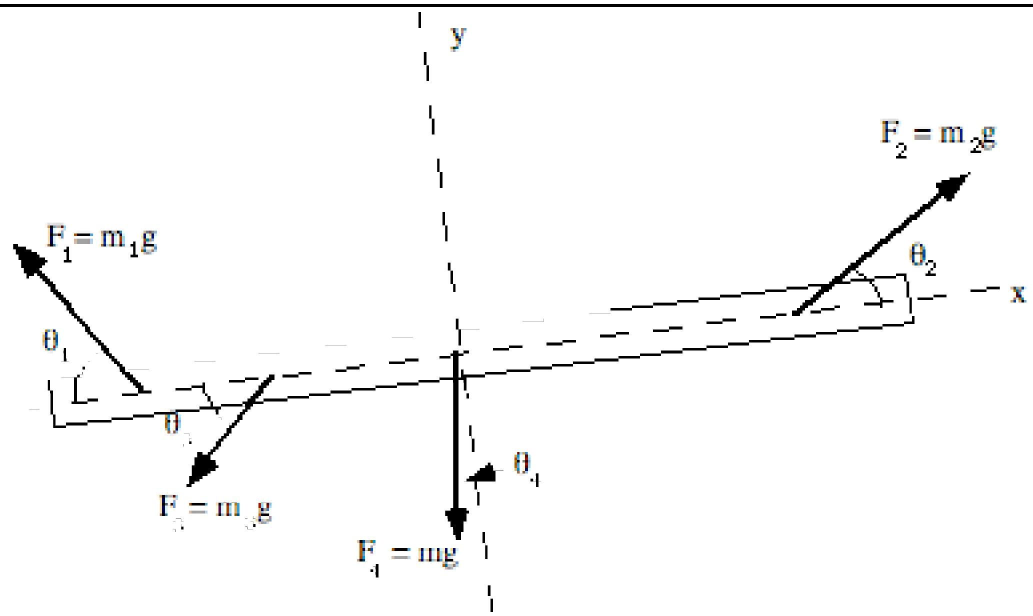

Figure 4

2. Record the mass responsible for each of the forces and the and direction of each force acting on the system. Also record the distance from the left end of the apparatus to the point at which each force acts.

3. The force of gravity acting on the apparatus acts at the center of gravity. The location of the center of gravity can be determined by finding the point at which you can balance the apparatus with a single force.

IV. Calculations and Analysis

1. On a page of graph paper add the forces acting on the apparatus by using a vector addition diagram, an example of which is given in Figure 5. **Note: your vector diagram will be the addition of the four forces F_1, F_2, F_3 and F_4. R** is know as the residual and is due to the error in the experiment.

2. Enter your data into an Excel spreadsheet. Figure 6 shows a partial spreadsheet for this experiment. It is up to you to complete this spreadsheet.

A + B + C = R

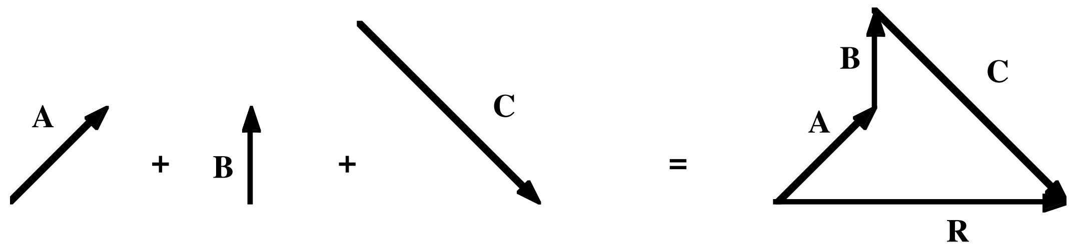

Figure 5

3. Now convert all directions of the forces to be with **<u>respect to the positive x-axis</u>** (e.g. angle 4 would be 270° - θ_4. By using angles with respect to the +x axis the signs of the components of the forces will be correctly calculated by the Excel's Sine and Cosine function. **Note:** it is **<u>not necessary</u>** that x be horizontal and y vertical! Refer to Figure 4.

4. Use Excel to resolve each mass into a force in Newtons. **Note** the multiplication by 0.001 in cell B8 which has the effect of converting grams to kilograms.

5. Then use Excel to resolve each force into its x and y components.

Note:

i) Excel use radians for angular measure in trigonometric calculations. Thus the multiplication of the angle in degrees by $\pi/180°$ in the formulas in cells C8 and D8.

ii) π in Excel is PI().

6. Then calculate the sums of the x and y components.

7. From the x and y components determine the magnitude and direction of the residual vector R (in cells D13 and D14). **Note:** the function ATAN2 returns an angle between $-\pi$ and $+\pi$.

	Mass (g)	Angle	Angle to +x	r (mm)
1				
2				
3				
4				
	Force (N)	Fx	Fy	
1	=9.81*B2*0.001	=B8*COS(D2*PI()/180)	=B8*SIN(D2*PI()/180)	
2				
3				
4				
	Sum	=SUM(C8:C11)		
	Magnitude of Residual			
	Direction of Residual		=ATAN2(C12,D12)*180/PI()	
	Torques (m•N)			
1	=D8*E2*0.001			
2				
3				
4				
Sum				
	Magnitude of Residual	from Vector Diagram ->		
	Direction of Residual	from Vector Diagram ->		
	Percent Difference in	Magnitude Residual ->		
	Percent Difference in	Direction Residual ->		

Figure 6

8. Determine the torque for each force about the left end of the system. **Important: Calculate all torques about the left end of the bar! By choosing the left end of the apparatus to take the torques about, positive torques will be due to forces with a positive y component and negative torques will be due to forces with a negative y component.** The factor of 0.001 in cell B17 is needed to convert mm to m.

9. Then calculate the sums of the torques.

10. Compare the residual found in step 1 to that found in step 7 by calculating the unbiased percent difference between the values of the magnitude and directions of the residual.

Note:

i) the unbiased percent difference between two experimental numbers n_1 and n_2 is given by:

$$\text{unbiased percent difference} = \left| \frac{n_1 - n_2}{\left(\frac{n_1 + n_2}{2} \right)} \right| \times 100\%$$

ii) Excel has an absolute value function. Be careful when you enter the formula for unbiased percent difference. The use of parentheses is recommended.

11. Would you expect the torques to have the same values individually if a different point were chosen as your axis? Would you expect the sum to differ?

12. What do you consider to be the sources of error in determining the magnitudes and directions of the individual forces involved in the experiment?

13. What do you consider to be the sources of error in determining the values of the torques in this experiment?

V. Questions

1. Three vectors ($\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$) are added to give a fourth vector $\vec{\mathbf{D}}$ ($\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$). The vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ all have magnitude of 2 Newtons..

   a) Can the magnitude of $\vec{\mathbf{D}}$ also be 2 Newtons? **Explain!**

   b) Can the magnitude of $\vec{\mathbf{D}}$ be zero? **Explain!**

2. If the component of a vector $\vec{\mathbf{D}}$ along the direction of another vector $\vec{\mathbf{A}}$ is zero, what can you conclude about the two vectors?

3. Under what circumstances does a vector lying in the xy plane have equal x and y components?