Students do not overcome conceptual difficulties after solving 1000 traditional problems

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(Received 28 March 1998; accepted 24 August 2001)

The relation between traditional physics textbook problem solving and conceptual understanding was investigated. The number of problems a student solved, as estimated by students themselves, ranged from 300 to 2900 with an average of about 1500. The students did not have much difficulty in using physics formulas and mathematics. However, we found that they still had many of the well-known conceptual difficulties with basic mechanics, and there was little correlation between the number of problems solved and conceptual understanding. This result suggests that traditional problem solving has a limited effect on conceptual understanding.

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[DOI: 10.1119/1.1484151]

I. INTRODUCTION

Because a major goal of physics teaching and learning is problem solving, the solution of exercises and problems is a major component of most physics classes, both in high schools and universities. Recent research in physics education has demonstrated that many students retain fundamental conceptual difficulties, even after instruction. One possible explanation for this situation is that students “haven’t done enough problems.” In Korea, as a result of the particular procedures for admission to the university, the students in this study have solved many exercises and problems (an average of 1500) as part of their preparation. In this paper we investigate whether this problem solving eliminates the conceptual difficulties found by researchers elsewhere. The conceptual understanding of these students was investigated using qualitative questions about basic mechanics.

The nature of the students’ background in problem solving is described in Sec. II. We also give a brief description of the Korean educational system in relation to the university entrance examination, because this exam was the reason why these students solved so many problems. The method of investigation and our results are described, followed by a summary.

II. STUDENT BACKGROUND

The twenty-seven students, nine females and eighteen males, involved in this study were first-year students in the Physics Education Department of Seoul National University (SNU) in 1994. Korea has a national curriculum in elementary and secondary school and all students take the same courses from the first grade (age 6) through the tenth grade (age 15), which is the first year of high school. Starting with the second year of high school, students are divided into two groups. One group takes courses with an emphasis on social science and humanities and the other group on science.

To be a science or a math education major in college, a student must be near the top 2% in her/his high school. Every year, a national examination is given to students who want to enter a university or college. The number of students taking the exam is about two and one-half times the total number of students admitted to all universities and colleges in Korea. Physics is a part of this national examination, which consists of multiple choice questions. The grade from this exam and their grades during their high school years become part of the criteria used for admission. To enter SNU, a student must be near the top 2% in her/his high school.

Due to the competitive nature of university admission, students work many problems. A significant portion of their time is spent on problem solving in science and mathematics, both in and out of class, especially during the last year of high school. Students practice problem solving using commercially available workbooks. A workbook consists of three parts: summary of content, example problems, and practice problems. Usually example problems have solutions with explanations, and practice problems have short answers without explanations. A typical workbook has about 1000 practice problems. Figure 1 shows problems from a common workbook.

III. METHOD

The students were enrolled in the introductory physics course, which consisted of three one-hour lectures, one two-hour laboratory session, and one two-hour recitation session each week. Fundamentals of Physics by Halliday, Resnick, and Walker was the textbook for the course and the first 22 chapters were covered during the first semester. Most of the survey and the tests for this study were done during recitation hours. The survey and tests were in written form.

The first part of the study was to determine the level of preparation. Two tests and one questionnaire were used. The tests used were the mathematics test by Halloun and Hestenes and the Mechanics Baseline Test (MBT). These tests provided a common basis for comparison with students in previous studies. Both tests were translated into Korean by the researchers. Students were asked the following questions to determine the amount of problem solving done by students.
A driver stopped her car because she saw an object in front. From the moment of seeing the object the car was 900 ft away and took seven seconds to come to a complete stop. The car was going along a straight road and the speed was 20 mph. How much time did it take for the driver to stop on the brake from the moment of seeing the object? Assume that the acceleration was constant after the brake was applied.

An object with mass m slid down a distance s along an inclined plane. The angle between the incline and the horizon is θ. Find the work done by the gravitational force, by the normal force and by both the normal and gravitational forces respectively. Let g be the gravitational acceleration.

Fig. 1. Some of the mechanics problems from a typical workbook.

(1) How many hours in a semester and how many semesters of physics classes did you have during high school?
(2) Did you have any other form of physics instruction, such as an individual tutor, other than the ones from your high school?
(3) How many workbooks did you use to prepare for the university entrance exam? What fraction of each workbook did you work through?
(4) As you answer question 3, which one of the following describes most accurately the meaning of “doing a problem in a workbook”?
(a) I worked the problem myself. It could be either solving a new problem, or solving again the ones covered before.
(b) I watched carefully while someone else was solving the problem.
(c) “Doing a problem” could mean either (a) or (b) to me.

The second part of the study investigated the students’ conceptual difficulties. A set of tutorials developed at the University of Washington was translated into Korean by the researchers and used during recitations. This material contains several pretests that are given before any related material is covered in either lecture or recitation. These consist of problems that require conceptual understanding without or with a sophisticated level of mathematics. Because the questions ask that students explain their reasoning, the students’ understanding can be probed more deeply. For the students in this study, these tests were given at the beginning of every recitation. The tests were not necessarily given before the material was covered in the lecture, but was always given before the students did the related part in the tutorials. The students’ responses to these tests were analyzed to investigate the relation between the amount of their traditional problem solving and their conceptual understanding.

IV. THE LEVEL OF PREPARATION IN PROBLEM SOLVING

According to the responses to questions 1 and 2 of the survey, students involved in this study had 2.4 physics class hours per week for two years. They also had 2 additional hours per week for two or three weeks in other forms of physics instruction such as an individual tutor. Almost all students replied that solving problems meant (a) in question 4. Therefore when a student said he/she solved a problem, it means he/she worked the problem on his/her own. As mentioned, a typical workbook has about 1000 practice problems. If a student answered that he/she used two workbooks and solved about 80% of the problems, the number of problems he/she solved was estimated as 1600. The estimate for each student is listed in Table I. Although these numbers are self-reported, and thus could be over- or underestimates, they are consistent with our experience. Solving problems to prepare for examinations has been common in Korea for many years. One of the authors of this paper (EK) is 17 years older than these students and had a similar experience when she was preparing for her entrance exam. In addition to regular classes, she completed one workbook (about 1000 problems) twice, solving most of the problems on her own.

Students are identified by capital letters in the first column of Table I. Because there were twenty-seven students, the last two students are marked by Z1 and Z2. The gender (g), the number of problems solved (n), and the number of correct responses for the math test (math) and for the Mechanics Baseline Test (MBT) are listed for each student. The tested difficulties are labeled by A, B, and C in the top row. Three problems for each difficulty are labeled by small letters (a, b, and c) in the second row. These labels match the subsections in Sec. V, where we discuss students’ conceptual difficulties. The rest of Table I shows the students’ responses, which will also be discussed in Sec. V. The absence of a student on the day of a particular test is marked by a minus sign. Three students (students Y, Z1, Z2) were absent on the day the survey was done. Student W said he studied one book mostly for content and the other mostly for problems. Student X said she studied four books. Both W and X did not mention what fraction of each book was done.

The number of problems a student reported solving ranged from 300 to 2900 with an average of about 1500. For comparison, the textbook by Halliday, Resnick, and Walker has 1400 questions and 3400 problems, and the textbook by Sears, Zemansky, and Young has 700 questions and 1600 problems. It is reasonable to conclude that this group of students had a significant amount of practice in problem solving. Generally students tend to work on the mechanics part more than on other parts because it is at the beginning of the workbook. But the specific number of mechanics problems done by each student was not surveyed.

The math test has 33 questions and the mechanics baseline test has 26 questions. Reported scores ranged from 50% to 60% for the math test, which was given as a pretest to students enrolled in a calculus-based university introductory physics. The scores for the MBT ranged from 32% to 73%. The test has been given as a post-test to various groups of students including high school students and Harvard honors students. In the current study, both tests were given as pretests. In terms of the test scores, students were well prepared in both math and mechanics. The average score for the math test was 30.4 (92.2%), very high compared to the reported scores. The average score for the mechanics baseline test was 16.6 (64%), which is on the high side of the reported scores.

V. CONCEPTUAL DIFFICULTIES

A significant number of the students had common conceptual difficulties. We categorize these into three areas: (A)
lack of differentiation among force, acceleration, and velocity, (B) misunderstanding Newton’s third law, and (C) a gap between the use of algebraic expressions and the associated physics concepts. As mentioned, this categorization is labeled by capital letter A, B, C in Table I. The problems or contexts used in each area are labeled by lower case letters (a, b, and c). If a student’s response was satisfactory, it is denoted by s. If the response was common but unsatisfactory, it is denoted by u and discussed in this section. If there are several distinct types of unsatisfactory answers, numbers are attached such as u1 and u2. No mark means that the student’s response was incorrect and not clear enough to be categorized. These responses are not discussed here. This analysis was done independently by one of the authors (EK) and a teaching assistant, who was a Ph.D. student in physics education. The two people disagreed less than 10% of the time and resolved differences through discussion.

It is not difficult to see from Table I that there is little correlation between \( n \) (the number of solved practice problems) and \( N \) (students’ success on answering conceptual pre-tests questions) as shown in Fig. 2. A student might have

Table I. Results of tests and questionnaire. Each student is identified by a capital letter in the first column. Because there were twenty-seven students, the last two students were marked by Z1 and Z2. The gender (g), number of problems solved (n), number of correct items for the math test (math) and Mechanics Baseline Test (MBT), and response types are listed for each student. For each difficulty there were three problems. A satisfactory response is marked by s. A frequent but unsatisfactory response is marked by u. Responses such as u1 and u2 refer to several distinct types as explained in the text. A blank space means that the response was not clear enough to be categorized. The absence of a student was marked by a ~ sign. The last row shows the number of students whose response was correct (\( N' \)).

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Number of correct responses (\( N' \)) 19 | 0 | 18 | 3 | 6 | 12 | 0 | 10 | 12 | 80

~A Lack of differentiation among force, acceleration, and velocity.
~B Newton’s third law.
~C Gap between the physics concepts and the algebraic expressions.

A. Lack of differentiation among force, acceleration, and velocity

For many students, the concepts of velocity, acceleration, and force are vaguely related to something moving and are not clearly distinguished. For example, it has been reported that about 65% of students in a college physics course believe that motion implies the existence of a force in the direction of motion. Another example is the confusion between acceleration and velocity. Students in this study did not have much difficulty solving a problem regarding linear motion with constant acceleration, which will be discussed in Sec. V C 3. However the common confusion of force, acceleration, and velocity was observed in three different problems.

(a) Students were asked to draw arrows to show the direction and the magnitude of velocity and acceleration for a ball rolling up and down an inclined plane. The arrows for velo-
ity were easy for students, but the ones for acceleration were not. For example, student D had a correct drawing for acceleration but gave a wrong explanation, “Acceleration was constant because the sum of the forces was zero.” Figure 3(a) shows the acceleration vectors drawn by student T. This student explained that the direction of the acceleration was opposite to the direction of motion and the magnitude of the acceleration was the same as that of the velocity. Student N’s drawing indicated that the magnitude of the acceleration was constant, but the direction was the same as the direction of velocity. There were nineteen students whose response was correct in both drawing and explanation (column A-a of Table I).

(b) A similar difficulty was found when students were asked to draw arrows to show the acceleration of a car speeding up, while moving clockwise along an elliptical track. The car was initially at rest at point A. No one had a correct response for this problem. One-third of the students, marked by u in column A-b of Table I, interpreted the increasing speed as the increasing magnitude of acceleration. The drawings of these students show the magnitude of the acceleration increasing as the car speeded up [Fig. 3(b)]. There were three different types of answers for the direction of the acceleration drawn by students. In one type of answer, acceleration pointed toward the center of the ellipse as in Fig. 3(b). In other drawings, the direction was the same as for the velocity. In the third type the direction was perpendicular to the velocity arrows, which was the correct direction for a constant speed. Some of the students’ responses were based on their knowledge about uniform circular motion, where the direction of the acceleration is perpendicular to the velocity and points toward the center of the circle.

(c) The confusion between force and acceleration was found in the third problem where two carts collided elastically, one of them initially being at rest [Fig. 3(c)]. The arrows above the carts show the velocity of the carts before and after the collision. Students were asked to compare the average acceleration of the two carts during the time interval between the two instants shown. Three students (F, T, V) wrote that the magnitude of A’s acceleration was the same as that of B’s during collision because the magnitude of the forces were the same due to the action/reaction law. (column A-c of Table I). Two more (students C, S) said they were the same without explanation.

If the problems were relatively easy, many students had correct responses (A-a, A-c). But when problems probed deeper understanding, the rate of correct responses dropped to zero (A-b). In other words, all students had difficulty distinguishing acceleration from force or from velocity in one or more problems.

B. Newton’s third law

There have been reports about student difficulties in understanding what could be called a “passive force” that adjusts itself in magnitude in response to an applied force, such as tension or a normal force. Brown17 reported that 50%–95% of students in the final year of secondary education (17–18 year olds) failed to compare the magnitude of two forces that were action–reaction pairs. He also found that the failure rate would have been lower if students understood that force was an interaction between two bodies. It is essential to understand Newton’s third law to understand this interaction. Normal forces and tension are not exceptions. Three problems were used to investigate this difficulty.

(a) A block was placed on a frictionless incline and a person pushed the block horizontally to keep it from moving [Fig. 4(a)]. The first question asked students to draw a free-body diagram of the block. It was explained to students that
In the second problem two blocks were pushed by a hand on a horizontal frictionless plane. The mass of block A is smaller than mass of block B. (c) A simple situation testing the understanding of tension. The three blocks all have the same mass, and the students were asked to compare the tension at A and A’.

(d) A complex situation testing the understanding of tension. Students were asked to compare the tension at points P and Q in the string which has mass.

A free-body diagram shows all the forces acting on the block. Then, students were asked which forces would change in magnitude if the person stopped pushing. Among twelve students (44%) who had the correct free-body diagram, only two students (W, Y) recognized that the normal force would decrease as the force exerted by the person disappeared. Student C said the force between the block and the incline changed, but did not use the term normal force. Fourteen students (52%, marked by u in column B-a of Table I) wrote that the normal force did not change because it is \( mg \cos \theta \) and the gravitational force and the angle of the incline did not change. Five more students (E, J, N, P, Z1) wrote that nothing changed, but mentioned no specific force or gave no explanation.

An object on an inclined plane is a common problem and students are familiar with the formula \( N = mg \cos \theta \). Student responses suggested that they could probably solve traditional problems with inclined planes successfully. However, a little change in the problem revealed that they might not understand where \( N \) came from.

In the second problem two blocks were pushed by hand on a horizontal frictionless plane [Fig. 4(b)]. The mass of block A is smaller than the mass of block B. First, students were asked to draw the free-body diagram for each block. About half of the students experienced some confusion about which object and which forces should be in the diagram and which should not.\(^{18}\) For example, students included the force of A acting on B in the free-body diagrams for both A and B. Some diagrams (11%) showed a deeper misunderstanding such that the force exerted by the hand was also acting on block B. The hand was then removed so that there was no longer a horizontal force pushing on the blocks. Many students (41%, marked by u in column B-b of Table I) seemed to think there had to be an action–reaction pair whenever two objects were in contact, because an action–reaction pair, \( F_{AB} \) and \( F_{BA} \), was still drawn in their diagrams.

(c) Tension was considered in the last problem. As reported in a previous study, tension is difficult for students.\(^4\) In the situation drawn in Fig. 4(c), six students (22%) said the tension at A’ was twice the tension at A (marked by u1 in column B-c of Table I), and two (7%) said it was zero (marked by u2). There were twelve students with correct answers with correct explanations (marked by s in column B-c of Table I). Comparing tension at various points along a string with mass [Fig. 4(d)] was even more difficult and no one gave a proper answer.

In the typical physics problems that the students had experienced, tension is simply given as a force existing in the string and its magnitude is constant throughout the massless string. The direction is considered only at the point of contact between the string and the object, where the tension always points away from the object. Without much explanation about the reasons why tension is treated in this way, this information about tension is used in solving problems. As a result, students could not say anything other than rewrite the familiar piece of knowledge that tension was the same throughout the string.

C. Gap between the physics concepts and the algebraic expressions

In a typical physics class, concepts are often introduced by verbal or mathematical definitions. The actual procedure that is necessary to construct the concept is not specified, and few students are successful at making connections by themselves. As a result, there are often gaps between the scientific concepts and the algebraic expressions.

It has been reported that after introductory physics courses many first-year university students could recall the definition of acceleration, but could not use this definition to compare the acceleration of two moving objects.\(^{13,19}\) Similar results, regarding the work-energy and the impulse-momentum theorems, were reported by Lawson et al.\(^{20}\) They reported that student reasoning was based solely on the mathematical definition without understanding the way physical quantities are related.

In the current study, this lack of connection was observed in three different contexts; acceleration, work, and the work-energy and impulse-momentum theorems.

I. Acceleration

Students were asked to draw arrows showing the acceleration for a car moving along an elliptical track. The car was moving at constant speed in the first part of the problem and speeding up in the later part. No student did both parts correctly. Eleven students (41%) used the definition of acceleration either in words or in the equation, \( a = \Delta v / \Delta t \), but failed to use either properly (marked by u in column C-a of Table I). Some examples of these responses are given below. These examples share similar characteristics to those reported in a previous study.\(^21\)

The first example is Fig. 5(a), where arrows represent the acceleration vectors for the car with constant speed. Three students (E, P, Z1) responded this way. Figure 5(b) shows the correct velocity vectors that they used to produce the acceleration vectors. They obtained the acceleration at point B by drawing \( v_B - v_A \) [Fig. 5(c)]. Other examples are the six re-
The situation presented in Fig. 6(b) is a straightforward application of the work-energy and impulse-momentum theorems. Two carts are initially at rest on a frictionless and horizontal table. The same constant force $F$ is exerted on each of the carts as they travel between the two marks on the table.

Among twenty-seven students, five students (marked by s1 in column C-c of Table I) used both the work-energy theorem and the impulse-momentum theorem. Another method chosen by seven students was to use the formula $2as = v_f^2 - v_i^2$ to find $v_f$, and then to find the final momentum and the kinetic energy. Five students (marked by s2) obtained the correct result after lengthy derivations. This type of response indicated that students were quite comfortable using the formula for linear motion with constant acceleration. Two (marked by s3) started from $\Delta K.E. = F \cdot s$ and determined which momentum was bigger using $K.E. = p^2/2m$. Twelve students, in all, had correct responses.

The common mistake made by six students was to assume that the two carts traveled between the two marks in the same time (marked by u in column C-c of Table I). Although these students did not provide an explanation, one possible reason for assuming the same time might be the distraction caused by the fact that the two pucks traveled the same distance.

It is worth noting that only five students started the problem from the work-energy and impulse-momentum theorems, although all students learned them in their high school years. Even when the momentum and kinetic energy were explicitly asked for in the problem, most of the students did not use these theorems, showing the gap between having been taught the formulas and utilizing them.

It has been reported that student difficulties are not due to erratic performances or lack of available knowledge, but due to their deficiencies in interpreting the knowledge they have. As mentioned above, students involved in this study appeared to know definitions of physical quantities and have the mathematical skill to use the necessary formulas. But their conceptual understanding was deficient, which led them to improper use of their knowledge.

VI. SUMMARY

The relation between the number of traditional problems solved and conceptual understanding was investigated. Tests of basic math and mechanics used in previous studies were given to have a common basis for comparison. The conceptual difficulties in basic mechanics were investigated by asking for student explanations to qualitative test questions based on research about conceptual understanding. Because of the competitive university entrance examination,
students involved in this study solved large numbers of problems. The scores on a math test and on the Mechanics Baseline Test were high. However, the students’ understanding was limited, and there was little correlation between conceptual understanding and the number of solved problems.

According to their written explanations in response to the questions probing conceptual understanding, the students did not have much difficulty in using physics formulas or mathematics, which was not surprising considering their problem-solving experience. But common difficulties in understanding basic concepts of mechanics were observed.

The result of this investigation provides evidence for the limits of traditional problem solving. Although traditional problem solving is an important part of studying to understand physics concepts, some aspects of conceptual understanding might require other approaches.

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